1st Korean Workshop on Graph Theory

August 26–28, 2015

Organized by
Seog-jin Kim and Sang-il Oum.

KAIST Center for Mathematical Challenges (KAIST CMC).
Dept. of Mathematical Sciences, KAIST.

1st Korean Workshop on Graph Theory

	August 26	August 27	August 28
9:40		9:40–10:20 권영수 10:20–10:40 *Coffee Break*	9:40–10:20 김석진 10:20–10:40 *Coffee Break*
10:40		10:40–11:20 노유미 11:30–12:10 박보람	10:40–11:20 최정옥 11:30–12:10 김연진
12:10		12:10–14:00 *Lunch*	12:10–14:00 *Lunch*
14:00	14:00–14:40 엄상일 14:50–15:30 최일규 15:30–16:00 *Coffee Break*	14:00–14:40 방세정 14:50–15:30 이상준 15:30–15:50 *Coffee Break* 15:50–16:20 정지수	
16:00	16:00–16:40 김재훈 16:50–17:30 이준경 17:30–18:00 *Problem session* 18:00–20:00 *Dinner*	<정지수, *to 16:20*> 16:20–16:50 이강주 17:00–17:30 *Problem session* 18:00–21:00 *Banquet*	

Lecture Hall:

- August 26, 27: Room 1501 (공동강의실), Building E6-1 (자연과학동 수리과학과)
- August 28: Room 3435, Building E6-1

Invited speakers:

- Young Soo Kwon (권영수), Yeungnam Univ.
- Seog-Jin Kim (김석진), Konkuk Univ.
- Younjin Kim (김연진), KAIST.
- Jaehoon Kim (김재훈), Birmingham Univ.
- Yoomi Rho (노유미), Univ. of Incheon.
- Boram Park (박보람), Ajou Univ.
- Sejeong Bang (방세정), Yeungnam Univ.
- Sang-il Oum (엄상일), KAIST.
- Sang June Lee (이상준), Duksung Women' s Univ.
- Joonkyung Lee (이준경), Univ. of Oxford.
- Ilkyoo Choi (최일규), KAIST.
- Jeong-Ok Choi (최정옥), GIST.

Speakers of Contributed Talks:

- Kang Ju Lee (이강주), Seoul National University.
- Jisu Jeong (정지수), KAIST.

Table of Contents

Abstracts

AUGUST 26

Number of cliques in graphs with a forbidden subdivision

SANG-IL OUM (엄상일)
(joint work with Choongbum Lee)
KAIST

We prove that for all positive integers t, every n-vertex graph with no K_t-subdivision has at most $2^{50t}n$ cliques. We also prove that asymptotically, such graphs contain at most $2^{(5+o(1))t}n$ cliques, where $o(1)$ tends to zero as t tends to infinity. This strongly answers a question of D. Wood asking if the number of cliques in n-vertex graphs with no K_t-minor is at most $2^{ct}n$ for some constant c.

A step towards both Geelan's Conjecture and Gyárfás' Conjecture

ILKYOO CHOI (최일규)
(joint work with O-joung Kwon and Sang-il Oum)
KAIST

The clique number is a trivial lower bound for the chromatic number of a graph. Since Erdős showed the existence of graphs with arbitrarily high chromatic number and arbitrarily high girth (so clique number is 2), in general, the chromatic number of a graph cannot be upper bounded by a function of its clique number. A class of graphs is said to be χ-bounded if such a function exists.

Vertex-minor and pivot-minors are graph containment properties such as (induced) subgraphs, subdivisions, and minors. Geelan conjectured that for any fixed graph H, the class of graphs with no H-vertex-minor is χ-bounded. This conjecture was known to be true only for one graph (proved by Dvořák and Král), but recently Chudnovsky, Scott, and Seymour proved it for any cycle. We add another class of graphs for which Geelan's Conjecture is true, namely, fan graphs.

We also ask the following question of whether Geelan's Conjecture can be generalized to pivot-minors: for any fixed graph H, are the class of graphs with no H-pivot-minor χ-bounded? We give some positive evidence to this question by proving that it is true for all cycles, which is a strengthening of the aforementioned result by Chudnovsky, Scott, and Seymour. This result can also be viewed as a partial result of the last open conjecture among the three conjectures made by Gyárfás' in 1985.

Typical Structure of graphs with no induced even cycles

JAEHOON KIM (김재훈)
(joint work with Daniela Kühn, Deryk Osthus, and Timothy Townsend)
University of Birmingham

We determine, for all $k \geq 6$, the typical structure of graphs that do not contain an induced $2k$-cycle. This verifies a conjecture of Balogh and Butterfield. Surprisingly, the typical structure of such graphs is richer than that encountered in related results. The approach we take also yields an approximate result on the typical structure of graphs without an induced 8-cycle or without an induced 10-cycle.

Some Advances on Sidorenko's Conjecture

JOONKYUNG LEE (이준경)
(joint work with David Conlon, Jeong Han Kim, Choongbum Lee)
University of Oxford

A bipartite graph H is said to have Sidorenko's property if the probability that the uniform random mapping from $V(H)$ to the vertex set of any graph G is a homomorphism is at least the product of the probabilities that each edge of H is mapped into an edge in G. In this talk, I will give an overview of the known results and new approaches to attack Sidorenko' s conjecture. This is joint work with David Conlon, Jeong Han Kim, and Choongbum Lee.

AUGUST 27

Several aspects of a skew-morphism

YOUNG SOO KWON (권영수)
Yeungnam University

A skew-morphism of a group Γ is a generalization of a group automorphism of Γ. Furthermore it is related to a group which can be expressed as a product of Γ and a cyclic group. As a consequence, it is related a regular Cayley map on Γ. This presentation deals with several aspects of a skew-morphism and some properties of it. Also some open problems related to a skew-morphism will be given.

On isomorphism classes of generalized Fibonacci cubes

YOOMI RHO (노유미)
(joint work with Jernej Azarija, Sandi Klavžar, Jaehun Lee and Jay Pantone)
Incheon National University

The generalized Fibonacci cube $Q_d(f)$ is the subgraph of the d-cube Q_d induced on the set of all strings of length d that do not contain f as a substring. It is proved that if $Q_d(f) \cong Q_d(f')$, then $|f| = |f'|$ and that there exist pairs of strings f, f' such that $Q_d(f) \cong Q_d(f')$ holds, where $|f| \geq \frac{2}{3}(d+1)$ and f' cannot be obtained from f by its reversion or binary complementation. Strings f and f' with $|f| = |f'| = d-1$ for which $Q_d(f) \cong Q_d(f')$ holds are characterized.

Coloring Squares of Kneser Graphs

BORAM PARK (박보람)
(joint work with Seog-Jin Kim)
Ajou University

The Kneser graph $K(n, k)$ is the graph whose vertices are the k-elements subsets of an n-element set such that two vertices are adjacent if and only if the vertices (sets) are disjoint. The square G^2 of a graph G is the graph defined on $V(G)$ such that two vertices u and v are adjacent in G^2 if the distance between u and v is at most 2 in G.

The problem of computing the chromatic number of $K^2(n, k)$ originally posed by Füredi was introduced and discussed in 2004. It is an interesting graph coloring problem, and also it is related with intersecting family and constant weight code. In this talk, we will present recent new results on the chromatic number of the square of the Kneser graph.

Geometric distance-regular graphs

SEJEONG BANG (방세정)
Yeungnam University

For a distance-regular graph Γ, a *Delsarte clique* is a clique with size $1 - \frac{k}{\theta_{\min}}$ where k is the valency of Γ and $\theta_{\min}$ is the smallest eigenvalue of Γ. A non-complete distance-regular graph Γ is called *geometric* if there exists a set $\mathcal{C}$ of Delsarte cliques such that each edge of Γ lies in a unique clique in $\mathcal{C}$. In this talk, we consider how to determine distance-regular graphs are geometric or not.

On a phase transition of the random intersection graph: Supercritical region

SANG JUNE LEE (이상준)
(joint work with Jeong Han Kim and Joohan Na)
Duksung Women's University

When each vertex is assigned a set, the intersection graph generated by the sets is the graph in which two distinct vertices are joined by an edge if and only if their assigned sets have a nonempty intersection. An interval graph is an intersection graph generated by intervals in the real line. A chordal graph can be considered as an intersection graph generated by subtrees of a tree. In 1999, Karoński, Scheinerman and Singer-Cohen [Combin Probab Comput 8 (1999), 131–159] introduced a random intersection graph by taking random assigned sets. The random intersection graph $G(n, m; p)$ has n vertices and their assigned sets are chosen to be i.i.d. random subsets of a fixed set M of size m where each element of M belongs to each random subset with probability p, independently of all other elements in M. Fill, Scheinerman and Singer-Cohen [Random Struct Algorithms 16 (2000), 156–176] showed that the total variation between the random graph $G(n, m; p)$ and the Erdös-Rényi graph $G(n, \hat{p})$ tends to 0 if $m = n^{\alpha}$, $\alpha > 6$, where $\hat{p}$ is chosen so that the expected numbers of edges in the two graphs are the same. In this paper, it is proved that the total variation still tends to 0 whenever $m \gg n^4$. We believe that this is the best possible.

Maximum Matching Width: new characterizations and a fast algorithm for dominating set

JISU JEONG (정지수)
(joint work with Sigve Hortemo Sæther and Jan Arne Telle)
KAIST

Tree-width and branch-width are connectivity parameters of importance in algorithm design. In 2012, Vatshelle introduced a graph parameter, maximum matching width, defined by a branch-decomposition over the vertex set of a graph G, using the symmetric submodular function obtained by taking the size of a maximum matching of the bipartite graph crossing the cut.

Tree-width and branch-width have alternative definitions through intersections of subtrees of a tree, where tree-width focuses on vertices and branch-width focuses on edges. We show that maximum matching width combines both aspects, focusing on vertices and on edges. Based on this we prove that given a graph G and a branch-decomposition of maximum matching width k, we can solve Dominating Set Problem in time $O^*(8^k)$. This runtime beats $O^*(3^{tw(G)})$-time algorithm for tree-width whenever $tw(G) > 1.893k$.

Path-intersection matrix and applications to networks

Kang-Ju Lee (이강주)
(joint work with D. Kho, W. Kook, J. Lee, and J. Lee)
Seoul National University

For a network G, we introduce a non-singular symmetric matrix, called a *path-intersection matrix*, that will provide a new method for computing the ratio

$$\frac{k(G)}{k(G/ab)}$$

where $k(G)$ is the tree-number of G and G/ab is obtained from $G \cup ab$ by contracting the new edge ab. The quantities $k(G)/k(G/ab)$ appear as invariants in various networks such as effective conductance in electrical networks and an ingredient for information centrality in social networks.

August 28

Nine Dragon Tree Conjecture

Seog-Jin Kim (김석진)
Konkuk University

For a loopless multigraph G, the *fractional arboricity* $Arb(G)$ is the maximum of $\frac{|E(H)|}{|V(H)|-1}$ over all subgraphs H with at least two vertices. Generalizing the Nash-Williams Arboricity Theorem, the Nine Dragon Tree Conjecture asserts that if $Arb(G) \le k + \frac{d}{k+d+1}$, then G decomposes into $k+1$ forests with one having maximum degree at most d. In this talk, we introduce Nine Dragon Tree Conjecture and present recent results.

On a pursuit-evasion model without instantaneous movement

Jeong Ok Choi (최정옥)
(joint work with J. Georges, D. Mauro)
Gwangju Institute of Science and Technology

There is a large amount of literature devoted to the graph-theoretic modeling of pursuit-evasion games. Using such terms as cops, robbers, and watchmen, these works posit the movement of intruders and pursuers from one vertex to another, not necessarily along edges, with the respective purposes of evading the pursuers and and capturing the intruders. Various assumptions on the mobility and knowledge of the pusuers and intruders, as well as various definitions of capture, determine the graph-theoretic parameter of primary interest: the minimum number of pursuers needed to guarantee the capture of all intruders, regardless of the strategies invoked by the intruders for escape. In this paper, we analyze a model in which all parties move along edges from one vertex to another, and no party moves with infinite speed. Capture occurs if and only if an intruder is within distance one of a pursuer. We also extend the model to one of greater generality in which evaders may have edges as destinations.

Sparse spanning k-strong subdigraphs in k-strong tournaments

Younjin Kim (김연진)
(joint work with Dong Yeap Kang, Jaehoon Kim, and Geewon Suh)
KAIST

A directed graph is strong if for any ordered pair of vertices u, v there exists a directed path from u to v. A directed graph is k-strong if it remains strong whenever fewer than k vertices are removed. In 2009, Bang-Jensen asked whether every n-vertex k-strong tournament contains a spanning k-strong subdigraph with at most $kn + f(k)$ arcs for a function $f(k)$. In this talk, we prove that every n-vertex k-strong tournament contains a spanning k-strong subdigraph with at most $(k + o(1))n$ arcs.

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