CORRIGENDUM TO OUR PAPER "TESTING BRANCH-WIDTH"

SANG-IL OUM AND PAUL SEYMOUR

Abstract. We fix a minor mistake in the proof of Theorem 5 in our paper "S. Oum and P. Seymour, *Testing branch-width*, J. Combin. Theory Ser. B, 97 (2007), pp. 385–393".

Theorem 5 of [1] proves the following.

Theorem 5. *Let f be a connectivity function on 2^V. Then no loose f-tangle of order $k+1$ exists if and only if the branch-width of f is at most k.*

The original proof in the first draft had been written in terms of tangles. There we had provided an easy but indirect proof using the "tangle lemma" of Robertson and Seymour [2] to prove Theorem 5. As it was written in the acknowledgment of [1], we presented a direct proof suggested by one of the reviewers. But unfortunately a minor error was made. We thank Daniel Král for pointing this out.

In the last sentence of the second paragraph of the proof of Theorem 5 in page 388, the following argument was used. (The numbering is for the convenience here.)

(1) *So Z and $B \setminus Z$ are both k-branched*
(2) and therefore $Z \cup (B \setminus Z) = A \cup B$ is k-branched
(3) and we deduce $C \in \mathcal{B}'$.

Daniel Král pointed out that (2) is false if $f(A \cup B) > k$. We replace (2) by the following proof.

Proof replacing (2). Let us choose W such that $C \subseteq W \subseteq A \cup B$ and $f(W)$ is minimum. Then for every subset Y of $A \cup B$, we have

$$f(Y) + f(W) \geq f(Y \cap W) + f(Y \cup W) \geq f(Y \cap W) + f(W),$$

and therefore $f(Y \cap W) \leq k$ if $f(Y) \leq k$. Thus $Z \cap W$ and $(B \setminus Z) \cap W$ are both k-branched and therefore W is k-branched because $f(W) \leq f(C) \leq k$. □

References

[1] S. Oum and P. Seymour, *Testing branch-width*, J. Combin. Theory Ser. B, 97 (2007), pp. 385–393.

[2] N. Robertson and P. Seymour, *Graph minors. X. Obstructions to tree-decomposition*, J. Combin. Theory Ser. B, 52 (1991), pp. 153–190.

Department of Mathematical Sciences, KAIST, Daejeon, South Korea.
E-mail address: sangil@kaist.edu

Department of Mathematics, Princeton University, Princeton, New Jersey, USA

Date: January 5, 2009; revised April 17, 2016.

The first author was supported by the SRC Program of Korea Science and Engineering Foundation (KOSEF) grant funded by the Korea government (MEST) (No. R11-2007-035-01002-0).