On Choosability with Separation of Planar Graphs with Forbidden Cycles

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A list assignment L assigns each vertex v a list $L(v)$ of available colors.

A proper L-coloring is a function f where

- for each vertex v: $f(v) \in L(v)$
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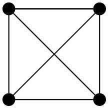

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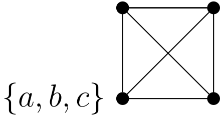

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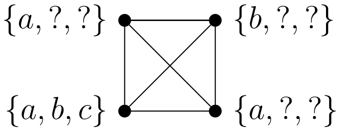

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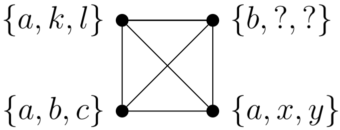

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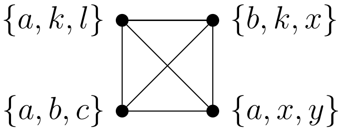

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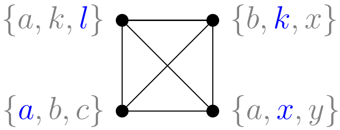

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Fact

Every graph is $(1, 0)$-choosable.
Every (k, d)-choosable graph is (k', d')-choosable for $k' \geq k$ and $d' \leq d$.

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For each $k < \chi_l(G)$, there is a threshold d where G is

- (k, d)-choosable
- but not $(k, d + 1)$-choosable.

Let $\chi_l(G, d)$ be the minimum k where G is (k, d)-choosable.

Theorem (Kratochvíl–Tuza–Voigt 1998)

$$\lim_{n\to\infty} \frac{\chi_l(K_n, 1)}{\sqrt{n}} = 1$$

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Conjecture (Füredi–Kostochka–Kumbhat 2013+)

For an n-vertex graph G,

$$\chi_l(G, d) \leq \chi_l(K_n, d)$$

Theorem (Thomassen 1994)

Every planar graph is 5-choosable.

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Notes	$k = 3$	d	$k = 4$	Notes
		0		
		1		
		2		
		3		
	—	4		

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Every planar graph with no 3-cycles is $(3, 1)$-choosable.

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Theorem (C.–Lidický–Stolee 2013+)

Every planar graph with no 5-, 6-cycles is $(3, 1)$-choosable.

Guaranteeing 3-choosability

Planar graphs with no 4-, i-, j-, 9-cycles for $i, j \in \{5, 6, 7, 8\}$.

3	4	5	6	7	8	9	authors	year
X		X		X		X	Alon–Tarsi	1992
X	X						Thomassen	1995
X		X	X				Lam–Shiu–Song	2005
X					X	X	Zhang–Xu–Sun	2006
X				X	X		Dvořák–Lidický–Škrekovski	2009
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If G is a plane graph with no 3-cycles and outer face F where $pq \in E(F)$, then G is L-colorable if L is a list assignment such that:

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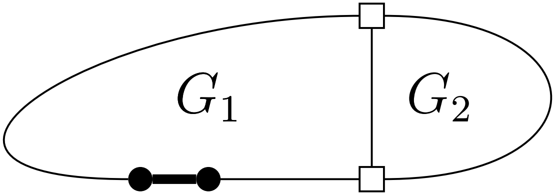

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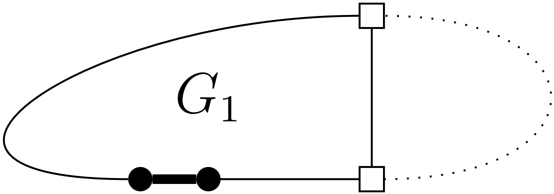

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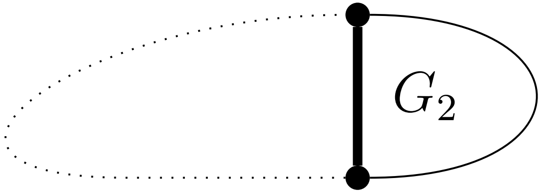

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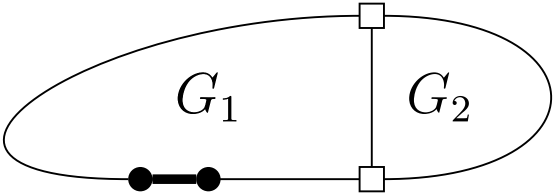

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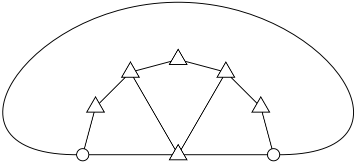

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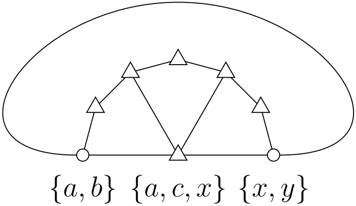


Theorem (C.–Lidický–Stolee 2013+)

If G is a plane graph with no 3-cycles and outer face F where $pq \in E(F)$, then G is L-colorable if L is a list assignment such that:

(*i*) $|L(v)| \geq 3$ *for* $v \in V(G) \setminus V(F)$,

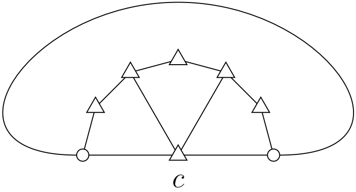
(*ii*) $|L(v)| \geq 2$ *for* $v \in V(F) \setminus \{p, q\}$,

(*iii*) $|L(v)| = 1$ *for* $v \in \{p, q\}$ *and* $L(p) \neq L(q)$,

(*iv*) *no two vertices with lists of size two are adjacent,*

(*v*) $|L(x) \cap L(y)| \leq 1$ *for each edge* xy.

Proof. First remove all unnecessary edges. There are no chords. The neighbors of v must be independent.



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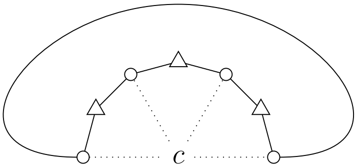
Proof. First remove all unnecessary edges. There are no chords. The neighbors of v must be independent.



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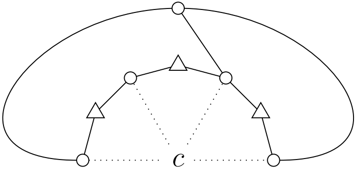
Proof. First remove all unnecessary edges. There are no chords. The neighbors of v must be independent.



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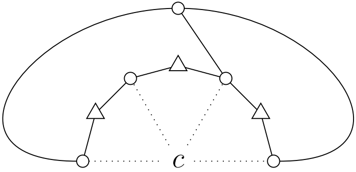
Proof. First remove all unnecessary edges. There are no chords.
The neighbors of v must be independent. There must be a good vertex.



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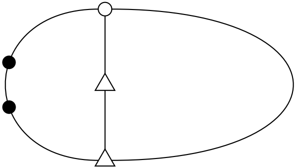
Proof. First remove all unnecessary edges. There are no chords.
The neighbors of v must be independent. There must be a good vertex.



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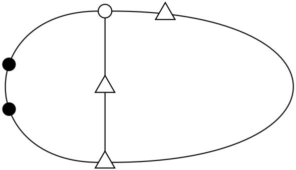
Proof. First remove all unnecessary edges. There are no chords. The neighbors of v must be independent. There must be a good vertex.



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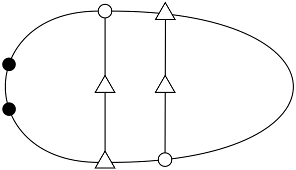
(iv) *no two vertices with lists of size two are adjacent,*

(v) $|L(x) \cap L(y)| \leq 1$ *for each edge* xy.

Proof. First remove all unnecessary edges. There are no chords.
The neighbors of v must be independent. There must be a good vertex.



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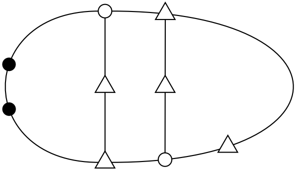
Proof. First remove all unnecessary edges. There are no chords.
The neighbors of v must be independent. There must be a good vertex.



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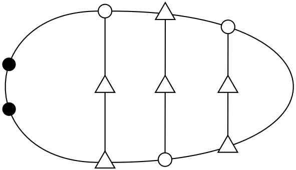
(*iv*) no two vertices with lists of size two are adjacent,

(*v*) $|L(x) \cap L(y)| \leq 1$ for each edge xy.

Proof. First remove all unnecessary edges. There are no chords.
The neighbors of v must be independent. There must be a good vertex.



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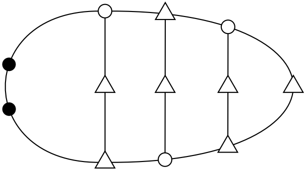
Proof. First remove all unnecessary edges. There are no chords.
The neighbors of v must be independent. There must be a good vertex.



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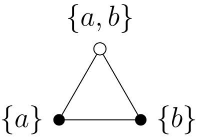
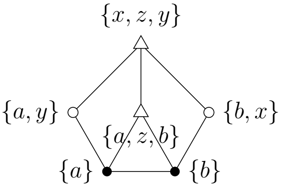
(*v*) $|L(x) \cap L(y)| \leq 1$ *for each edge* xy.

Proof. First remove all unnecessary edges. There are no chords.
The neighbors of v must be independent. There must be a good vertex.





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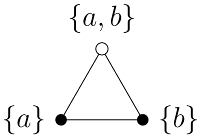
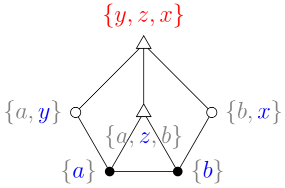
Proof. First remove all unnecessary edges. There are no chords.
The neighbors of v must be independent. There must be a good vertex.



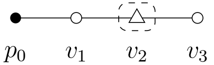


Theorem (C.–Lidický–Stolee 2013+)

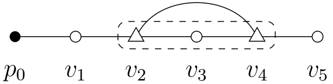
Every planar graph with no 4-cycles is $(3, 1)$*-choosable.*

Theorem (C.–Lidický–Stolee 2013+)

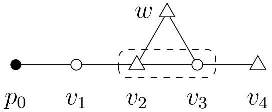
Every planar graph with no 4-cycles is $(3, 1)$-choosable.



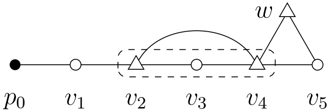
(X1a)



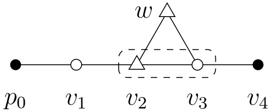
(X2)



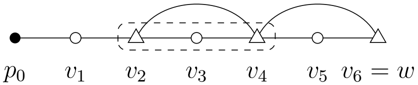
(X1b)



(X2)



(X1b)



(X2)

Theorem (C.–Lidický–Stolee 2013+)

Every planar graph with no 5-, 6-cycles is $(3, 1)$*-choosable.*

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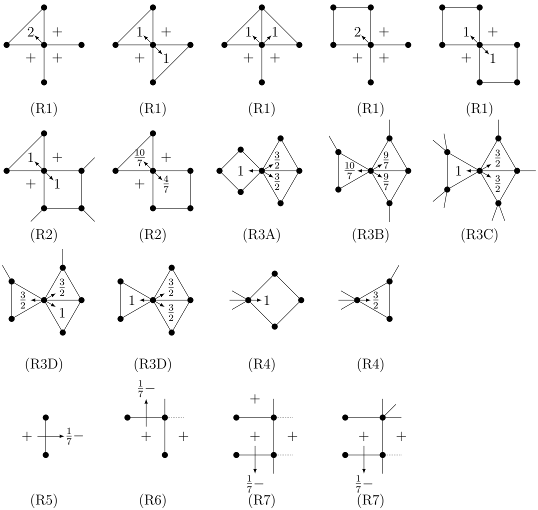

Notes	$k = 3$	d	$k = 4$	Notes
Trivial	Yes.	0	Yes.	Trivial
Yes for no 3-cycles				
Yes for no 4-cycles	?	1	Yes.	K–T–V, Škrekovski
Yes for no 5-, 6-cycles				
	No.	2	?	
	No.	3	No.	
	—	4	No.	

Question (Škrekovski 2001)

Are all planar graphs $(3, 1)$*-choosable?*

Question (Kratochvíl–Tuza–Voigt 1998)

Are all planar graphs $(4, 2)$*-choosable?*

Thank you for your attention!