3-coloring triangle-free planar graphs with a precolored 9-cycle

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December 21, 2013

A graph G is k-colorable if there is a function f where

- for each vertex v: $f(v) \in [k]$
- for each edge xy: $f(x) \neq f(y)$

A graph G is k-critical if

- G is not $(k-1)$-colorable
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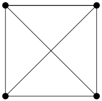

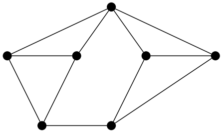

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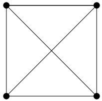

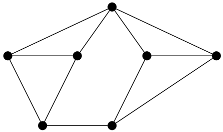

A graph G is C-critical for k-coloring if

- for each edge e, there is a k-coloring f_e of $V(C)$ where
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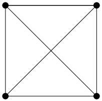

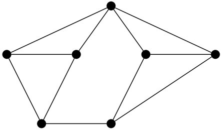

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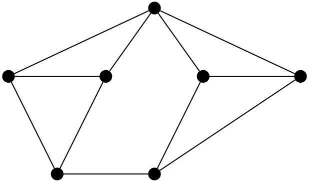

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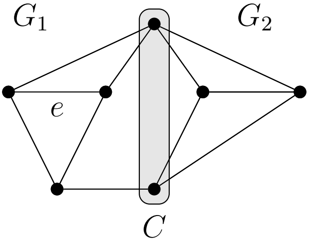
- not 3-colorable
- each subgraph is 3-colorable



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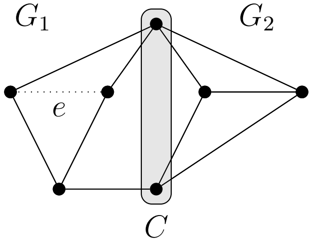

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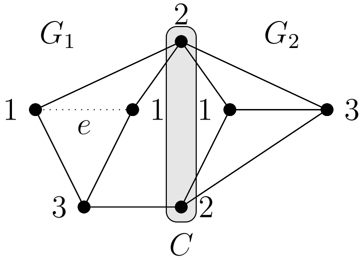

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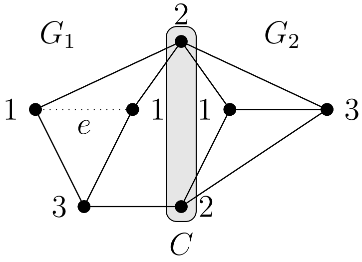
There exists a 3-coloring of $V(C)$ that extends to $G_1 - e$ but does not extend to G_1.



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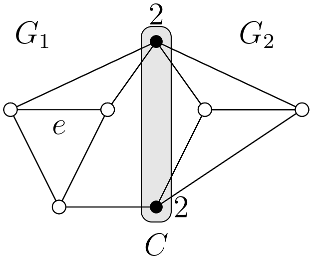

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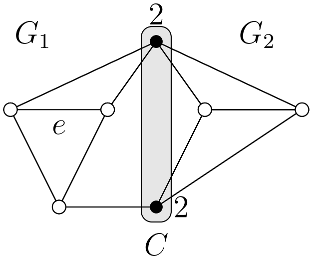

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A graph G is C-*critical* for k-coloring if for each $e \in E(G)$, there exists a k-coloring f_e of $V(C)$ that extends to $G - e$ but does not extend to G.

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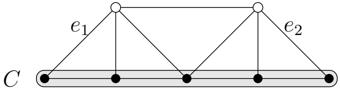

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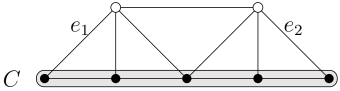

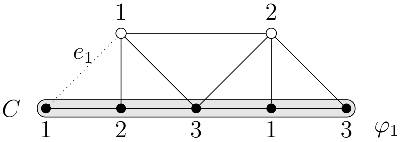

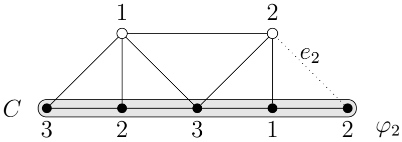

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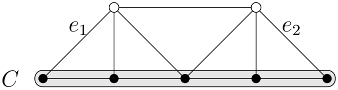

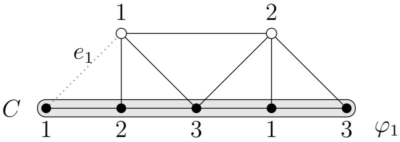

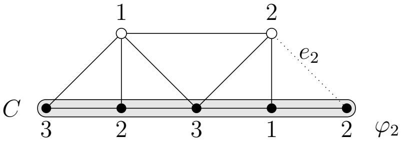

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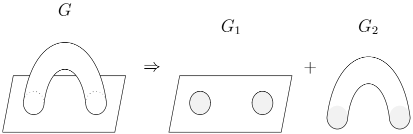

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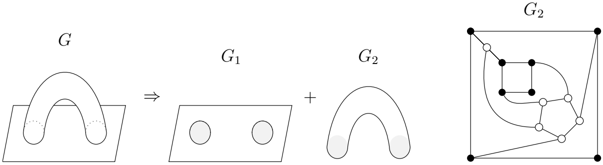

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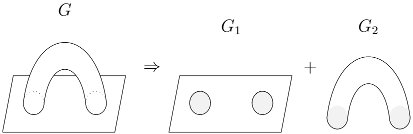
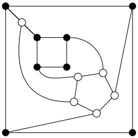


G_2



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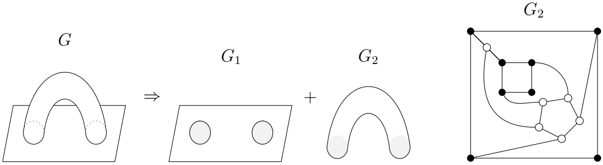

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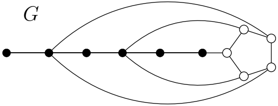

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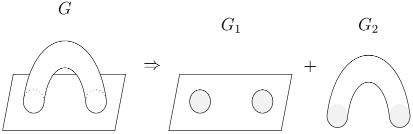

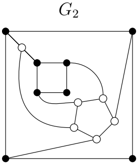

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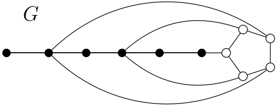

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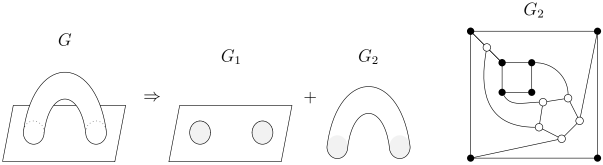

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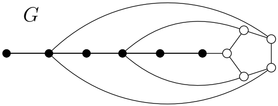

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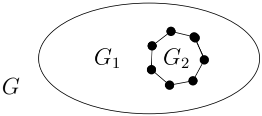

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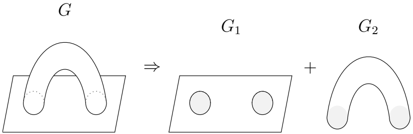

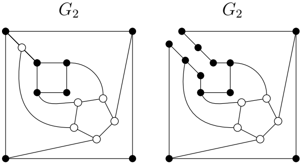

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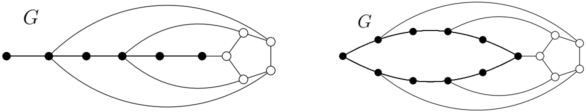

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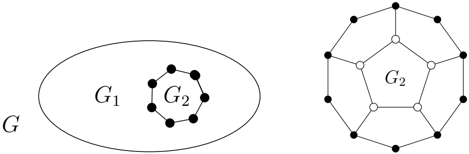

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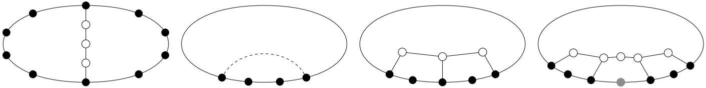

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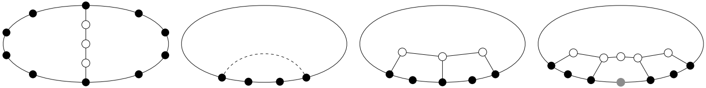

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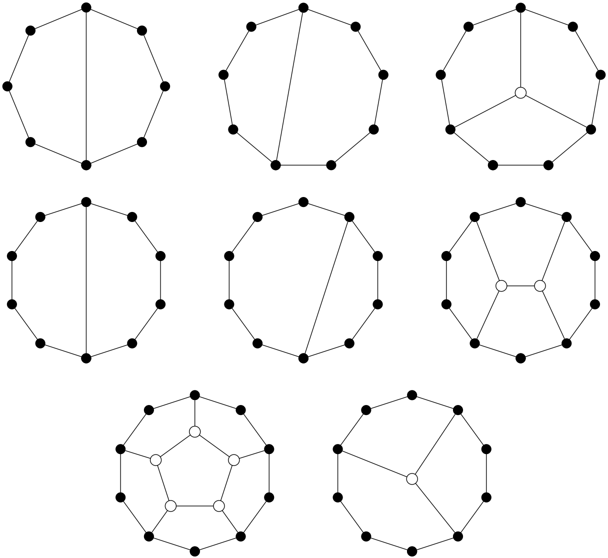
- $|C| \leq 16$ by Dvořák–Lidický 2013+

$|C| \leq 10$



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Known characterizations:

- $|C| \in \{4, 5\}$ by Aksenov 1974
- $|C| = 6$ by Gimbel–Thomassen 1997
- $|C| = 6$ by Aksenov–Borodin–Glebov 2003
- $|C| = 7$ by Aksenov–Borodin–Glebov 2004
- $|C| = 8$ by Dvořák–Lidický 2013+
- $|C| = 9$ by C.–Ekstein–Holub–Lidický 2014+

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"nice" plane graph: has no separating 4-cycles or 5-cycles.

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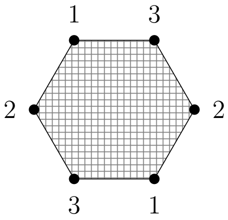
Theorem (Gimbel–Thomassen 1997, Aksenov–Borodin–Glebov 2003)

If G is a "nice" plane graph of girth 4 bounded by a cycle C of length 6, then G is C-critical if and only if G "looks like" below.



Theorem (Aksenov–Borodin–Glebov 2004)

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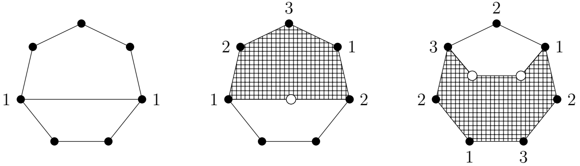

Theorem (Dvořák–Lidický 2013+)

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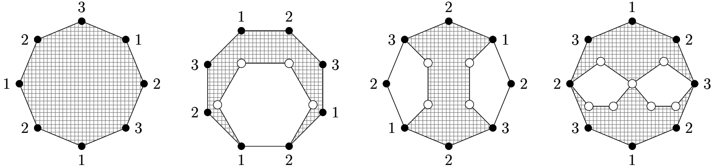

Theorem (C.–Ekstein–Holub–Lidický 2014+)

If G is a "nice" plane graph of girth 4 bounded by a cycle C of length 9, then G is C-critical if and only if G "looks like" a graph below (2 more).

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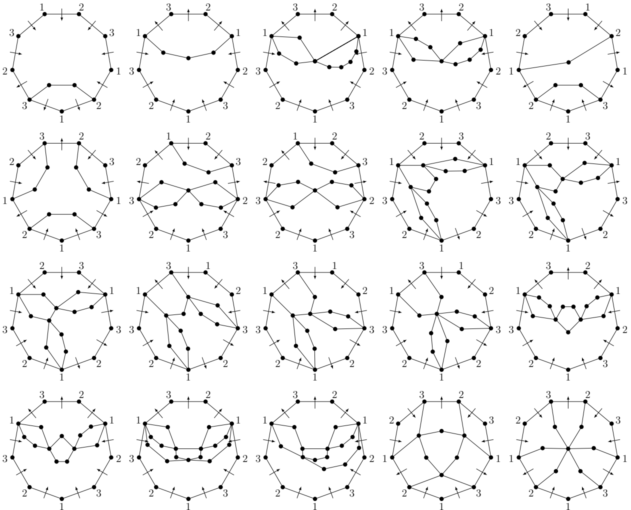

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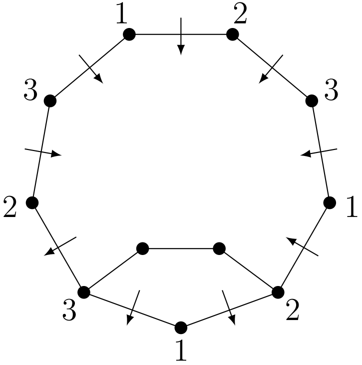

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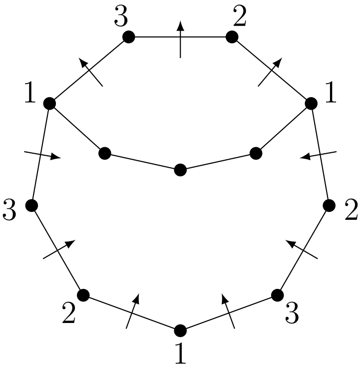

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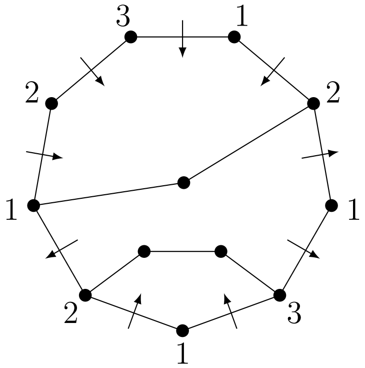

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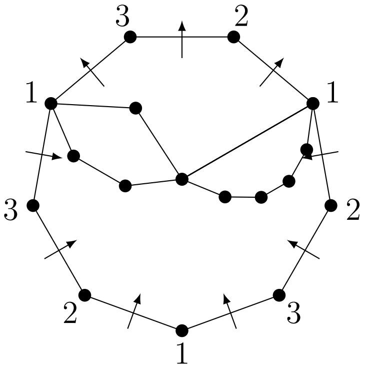

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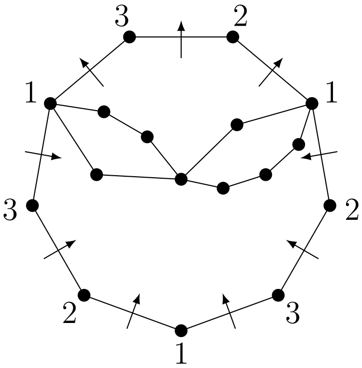

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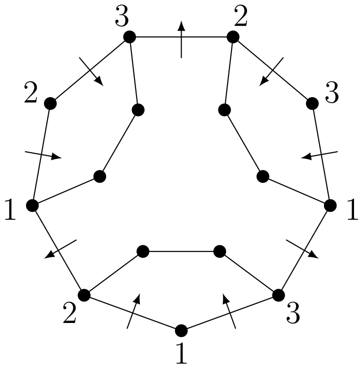

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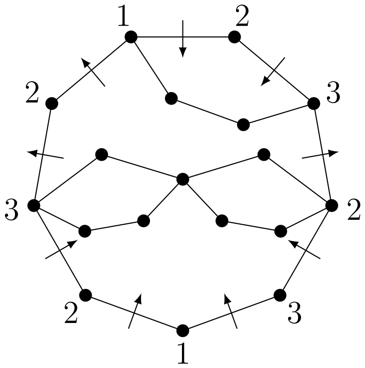

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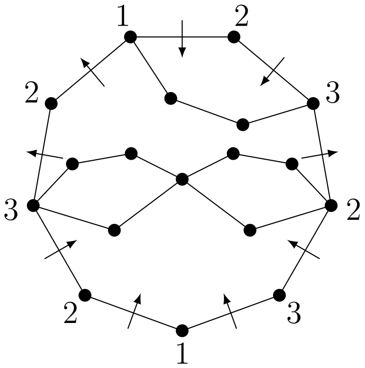

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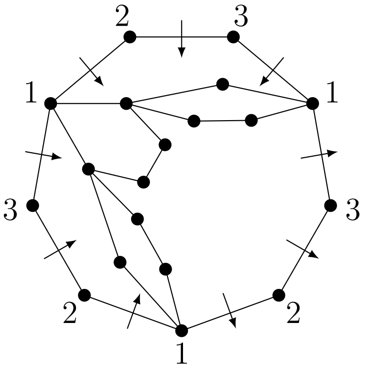

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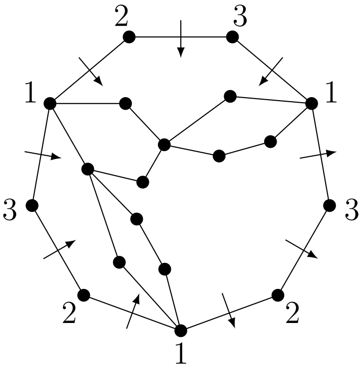

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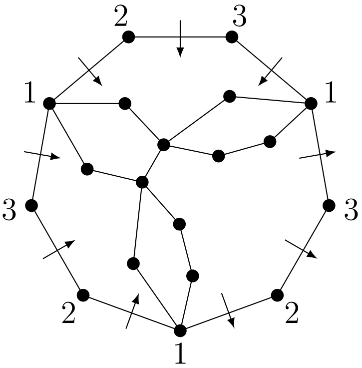

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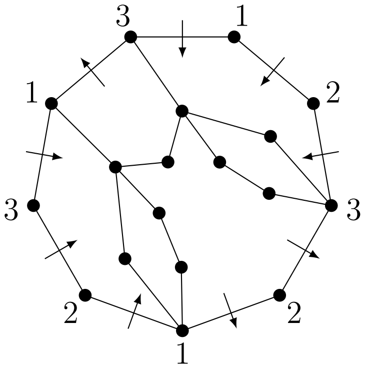

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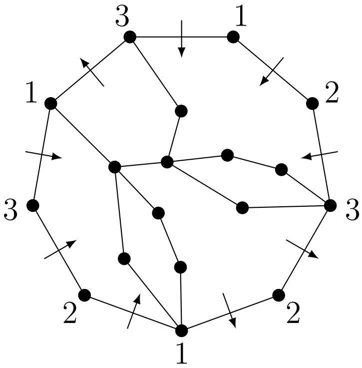

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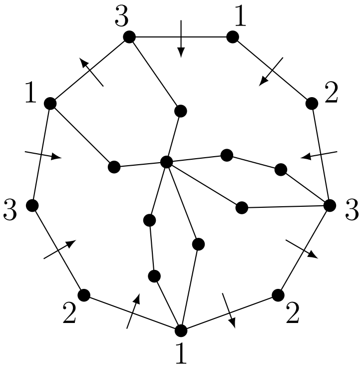

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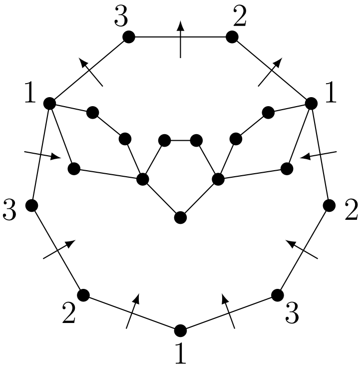

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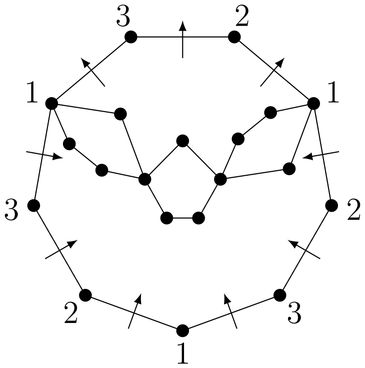

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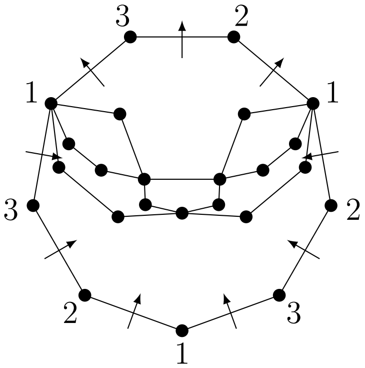

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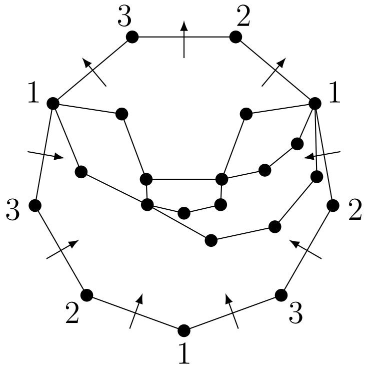

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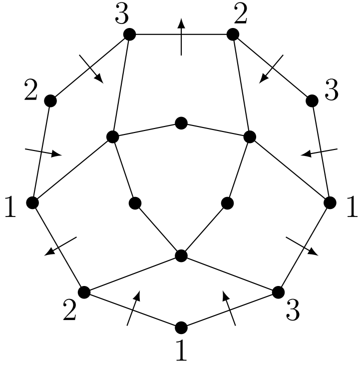

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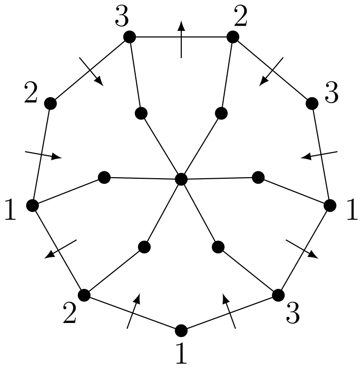

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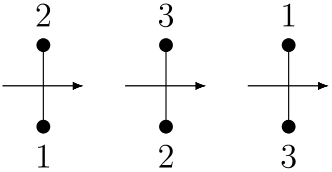

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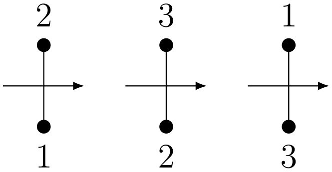
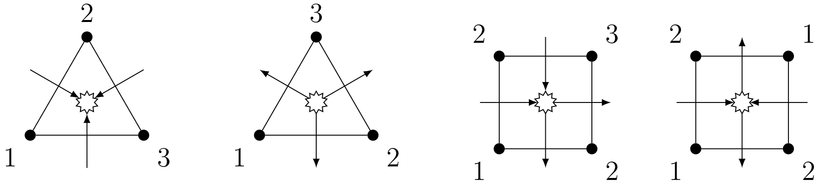


(In-edges - out-edges) of every face is a multiple of 3!



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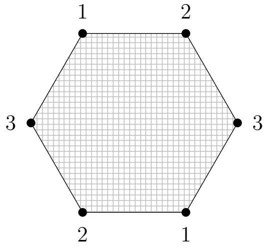

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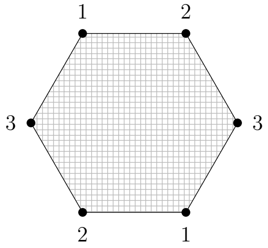

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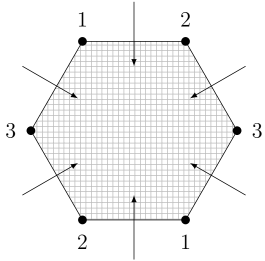

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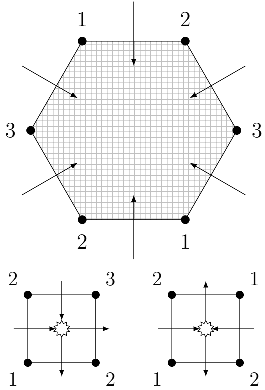

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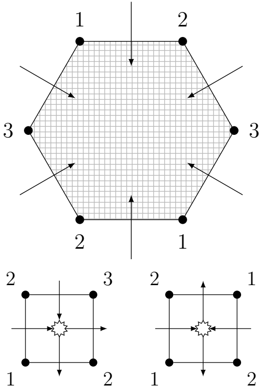

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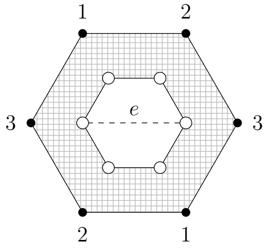

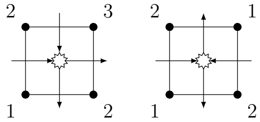

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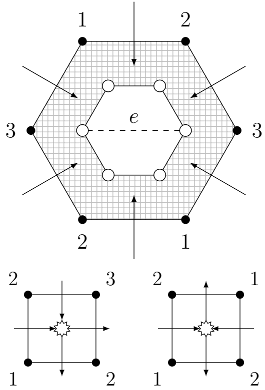

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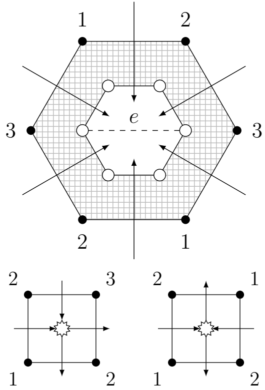

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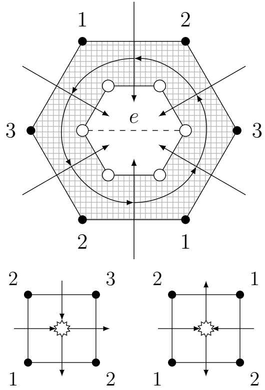

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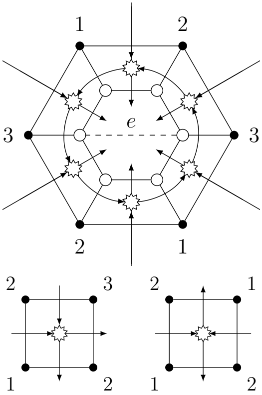

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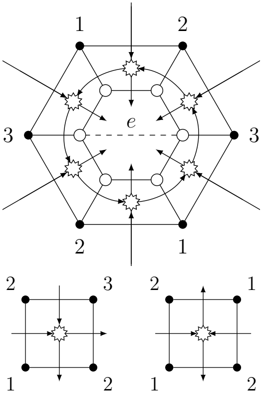

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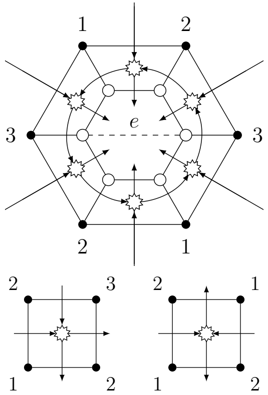

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Corollary (Dvořák–Kráľ–Thomas 2014+)

If G is a "nice" plane graph of girth 4 bounded by a cycle C of length c and is C-critical, then

$$\begin{aligned} c = 6 &: \quad \emptyset \\ c = 7 &: \quad \{5\} \\ c = 8 &: \quad \emptyset, \{6\}, \{5, 5\} \\ c = 9 &: \quad \{7\}, \{5, 6\}, \{5, 5, 5\}, \{5\} \end{aligned}$$

are the only possible multisets of faces of length at least 5.

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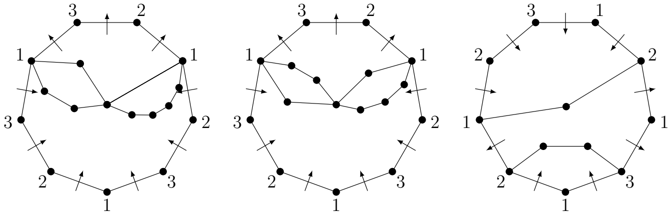

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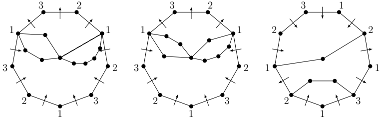

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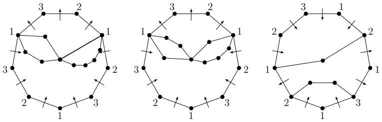

Proof: $(\Rightarrow)$

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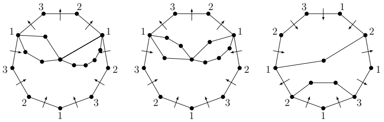

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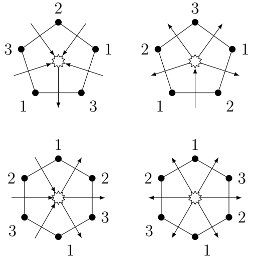

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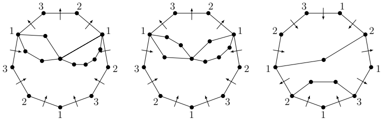

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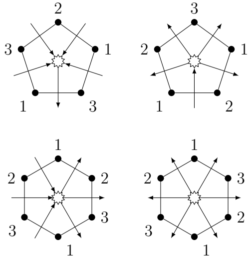

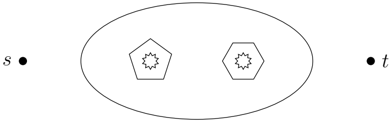

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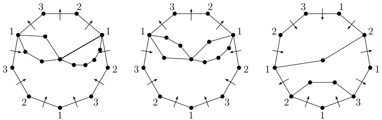

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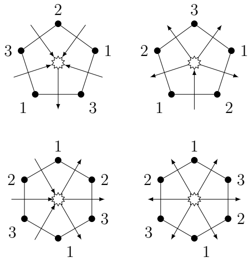

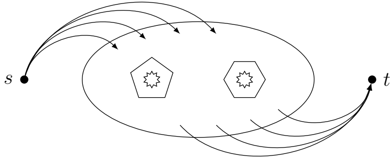

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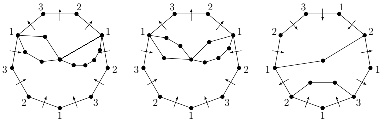

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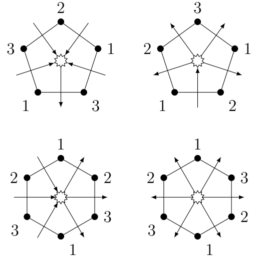

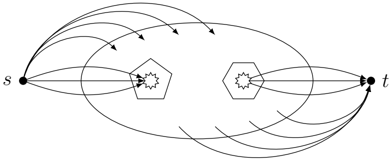

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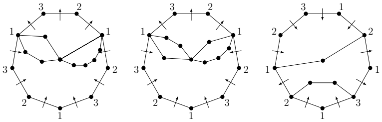

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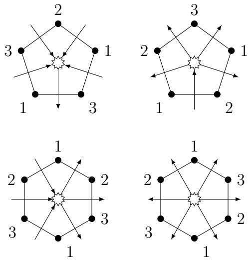

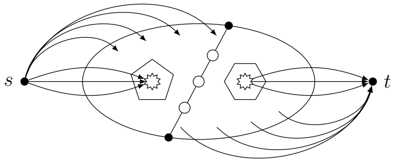

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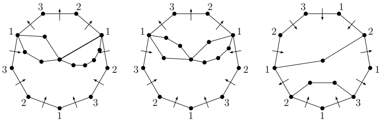

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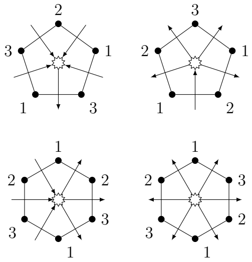

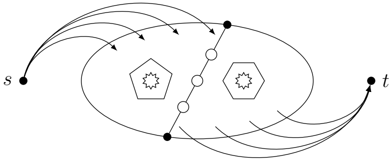

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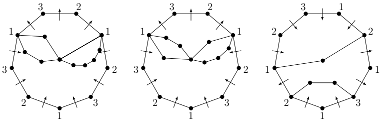

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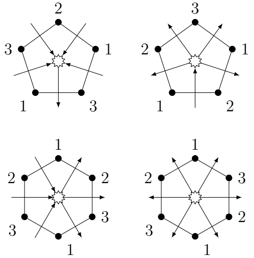

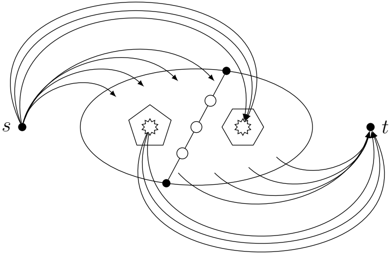

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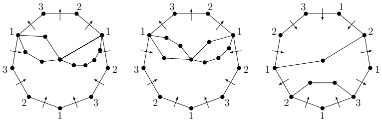

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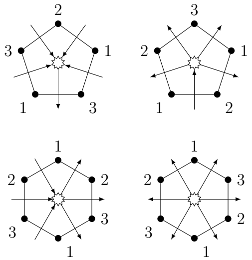

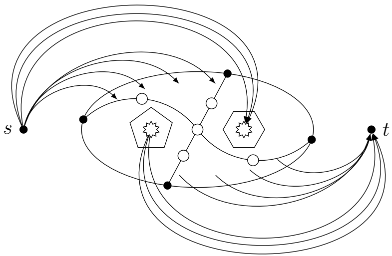

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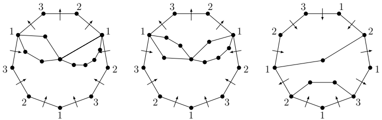

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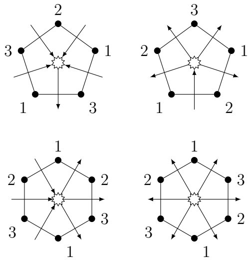

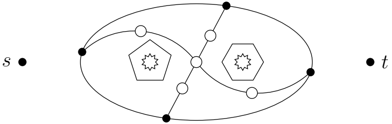

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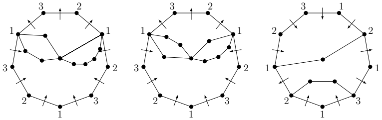

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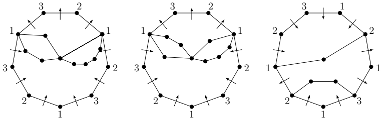

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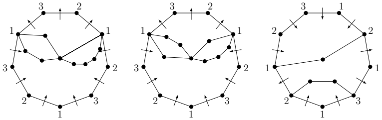

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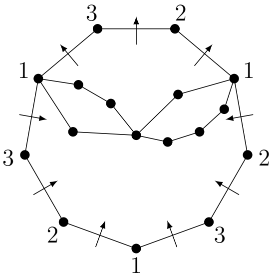

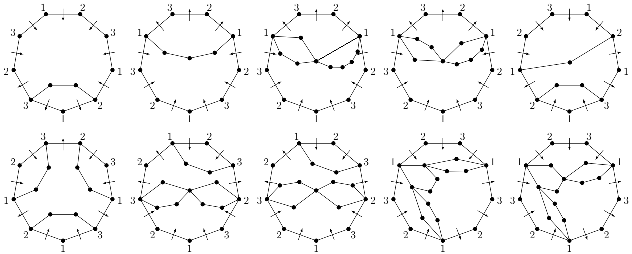

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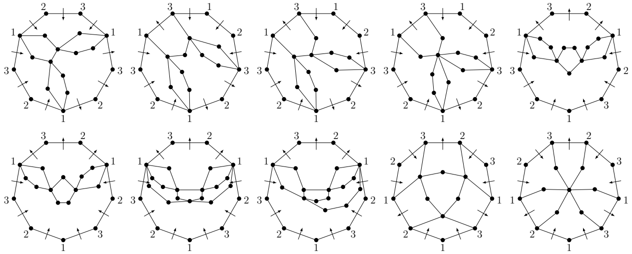

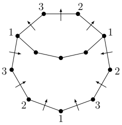

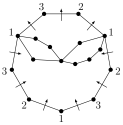

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