12th Korea-Japan Workshop on Algebra and Combinatorics

Thursday

09:30	-	Takao Komatsu:	*a-, q-, λ- generalization of poly-Bernoulli numbers and poly-Cauchy numbers.*
10:30	-	Etsuko Bannai:	*Tight relative t-designs on Q-polynomial association schemes.*
11:30	-	Jongyook Park:	*Bounds on eigenvalues for distance-regular graphs.*
12:00	-	LUNCH	
14:00	-	Michel Pocchiola:	*Arrangements of double pseudolines.*
15:00	-	Michiaki Onodera:	*Geometric flows for quadrature surfaces.*
16:00	-	COFFEE BREAK	
16:30	-	O-joung Kwon:	*Unavoidable vertex-minors in large prime graphs.*
17:00	-	Tatsuro Ito:	*The classification of TD-pairs of Type II.*

Friday

10:00	-	Jang Soo Kim:	*The Selberg integral and Young books.*
11:00	-	Vincent Pilaud:	*Signed tree associahedra.*
12:00	-	LUNCH	
14:30	-	Suyoung Choi:	*Enumeration of toric objects and wedge operation on simplicial complexes.*
15:30	-	Boram Park:	*Chromatic-choosability of the power of graphs.*
16:00	-	COFFEE BREAK	
16:30	-	Michael Dobbins:	*Epsilon net and Helly type theorems for unit balls and k-flat transversals.*
17:00	-	Jeongok Choi:	*Decomposition of Regular Hypergraphs.*

Saturday

09:30	-	Alexander Gavrilyuk:	*Characterization of the bilinear forms graphs $Bil_2(n,n)$, $n > 2$, by their intersection arrays.*
10:30	-	Ferenc Szöllősi:	*Nonexistence results for complex equiangular tight frames.*
11:00	-	Yoshio Sano:	*Travel groupoids on graphs.*
12:00	-	LUNCH	

ABSTRACTS

- **Etsuko Bannai** (at Shanghai)

  Title: *Tight relative t-designs on Q-polynomial association schemes.*

  Abstract: The concepts of t-design and relative t-design are defined on Q-polynomial association schemes by P. Delsarte. Euclidean t-design is defined as a generalization of spherical t-design. During the studies of Euclidean designs, we found similarities between relative t-designs on Q-polynomial association schemes and Euclidean t-designs. We give the definition of relative t-design on Q-polynomial scheme in the new style, which is similar as Euclidean t-design. The Fisher type lower bounds for the cardinalities of relative $2e$-design are known in terms of the dimension of the column space of primitive idempotents of the given Q-polynomial schemes. However the explicit formulas are very difficult to obtain for general cases. The works by Z. Xiang, B-B-Suda-Tanaka shows that in the following cases we have the explicit formula for the Fisher type bounds: the Hamming schemes $H(n, q)$, Johnson schemes $J(n, d)$, and P-and Q-polynomial schemes with some additional conditions. In this talk we consider the tight relative 2-desings on 2 shells in $H(n, 2)$ and $J(n, d)$ with small d. These are joint work with Eiichi Bannai, Hideo Bannai and Y. Zhu.

- **Jeongok Choi** (GIST)

  Title: *Decomposition of Regular Hypergraphs.*

  Abstract: An *r-block* is a $0, 1$-matrix in which every row has sum r. Let S_n be the set of pairs (k, l) such that the columns of any $(k + l)$-block with n rows split into a k-block and an l-block. We determine S_n for $n \leq 5$. In particular, $S_3 = \{(k, l) : 2|kl\}$, $S_4 = \{(k, l) : (6|k \text{ or } l) \text{ and } k, l > 1\}$, and $S_5 = \{(k, l) : 11 \neq \min\{k, l\} > 7$ and each value in $\{3, 4, 5\}$ divides k or $l\}$. The problem arose from a list-coloring problem in digraphs and is a refinement of the notion of indecomposable hypergraphs. This is joint work with D. B. West.

- **Suyoung Choi** (Ajou University)

  Title: *Enumeration of toric objects and wedge operation on simplicial complexes.*

  Abstract: A fundamental result of toric geometry is that there is a bijection between toric varieties and fans. More generally, it is known that some classes of manifolds having well-behaved torus actions can be classified in terms of combinatorial data containing simplicial complexes with m vertices. In this talk, we investigate the relationship between toric objects over a simplicial complex K and those over the complex obtained by simplicial wedge operations from K. In particular, we classify and enumerate small covers over a simplicial complex with a few vertices.

- **Michael Dobbins** (POSTECH)

  Title: *Epsilon net and Helly type theorems for unit balls and k-flat transversals.*

  Abstract: In this talk I present a weak epsilon net theorem for families of k-flats in d dimensions intersecting unit balls. This with a related fractional Helly type theorem implies a (p, q)-piercing theorem for unit balls with k-flat transversals. This is joint work with Andreas Holmsen.

- **Alexander Gavrilyuk** (Tohoku University)

  Title: *Characterization of the bilinear forms graphs $Bil_2(n, n)$, $n > 2$, by their intersection arrays.*

  Abstract: The bilinear forms graph $Bil_q(n, m)$ is a graph defined on the set of $n \times m$-matrices over the finite field F_q, with two matrices A, B being adjacent if $\mathrm{rank}(A - B) = 1$.

  These graphs are distance-transitive and Q-polynomial. In 1999, K. Metsch showed that a distance-regular graph with intersection array of $Bil_q(n, m)$ is indeed $Bil_q(n, m)$, if ($q = 2$ and $m \geq n + 4$) or ($q \geq 3$ and $m \geq n + 3$). Thus the open cases are ($q = 2$ and $m \in \{n, n + 1, n + 2, n + 3\}$) and ($q \geq 3$ and $m \in \{n, n + 1, n + 2\}$).

  In this work, we show that $Bil_2(n, n)$ is characterized by its intersection array. This is joint work with J. H. Koolen

- **Tatsuro Ito** (Kanazawa University)

  Title: *The classification of TD-pairs of Type II.*

  Abstract: A generalization of the Onsager algebra is introduced and its finite-dimensional irreducible representations are determined. As a result, TD-pairs of Type II are classified.

- **Jang Soo Kim** (KIAS)

  Title: *The Selberg integral and Young books.*

  Abstract: The Selberg integral is an important integral first evaluated by Selberg in 1944. Stanley found a combinatorial interpretation of the Selberg integral in terms of probability. In this talk we find a connection between the Selberg integral and shifted Young tableaux of staircase shape and standard Young tableaux of square shape. We define 'Young books' which are a generalization of both of these shifted and standard Young tableaux and find a formula for the number of Young books. This is joint work with Suho Oh.

- **Takao Komatsu** (Hirosaki University)

  Title: *a-, q-, λ- generalization of poly-Bernoulli numbers and poly-Cauchy numbers.*

  Abstract: The concept of poly-Bernoulli numbers is a kind of generalizations of the classical Bernoulli numbers. Similarly, that of the poly-Cauchy numbers are of the classical Cauchy numbers. We give varieties directions of further generalization of both numbers. We also show their several combinatorial and arithmetical properties.

- **O-joung Kwon** (KAIST)

  Title: *Unavoidable vertex-minors in large prime graphs.*

  Abstract: We say a graph is prime if it cannot be decomposable with respect to a certain "split" operation. It is known that if a graph G can be decomposed into a split and H is a vertex-minor of G, then H can be obtained from only one part of the split. So, it is valuable to observe what vertex-minors exist in prime graphs having some property. Here, we prove Ramsey type theorem for prime graphs with vertex-minor operation.

  In more detail, we prove that for each n, there exists an N such that every prime graph on at least N vertices contains a vertex-minor isomorphic to either a cycle of length n or a graph consisting of two disjoint cliques of size n joined by a matching.

  We will describe two big steps of the proof. We also plan to provide a main tool, which is called a blocking sequence in a graph, and describe how to use it. And we will pose some open problems behind this result.

  This is joint work with Sang-il Oum.

- **Michiaki Onodera** (Kyushu University)

  Title: *Geometric flows for quadrature surfaces.*

  Abstract: For prescribed points and weights, a quadrature surface is an (N-1)-dimensional closed hypersurface which admits a quadrature formula, namely that the surface integral of any harmonic polynomial on the surface has the same value as the weighted sum of the values of the polynomial at the prescribed points. We show that a family of quadrature surfaces corresponding to the varying weights forms a geometric flow which can be described by an evolution equation. This characterization enables us to study quadrature surfaces through the investigation of the flow. It is proved that the flow is uniquely solvable under the geometric condition that the initial surface has positive mean curvature. As a consequence, a bifurcation criterion for quadrature surfaces is obtained.

- **Boram Park** (NIMS)

  Title: *Chromatic-choosability of the power of graphs.*

  Abstract: The kth power G^k of a graph G is the graph defined on $V(G)$ such that two vertices u and v are adjacent in G^k if the distance between u and v in G is at most k. Let $\chi(H)$ and $\chi_l(H)$ be the chromatic number and the list chromatic number of H, respectively. A graph H is called *chromatic-choosable* if $\chi_l(H) = \chi(H)$. It is an interesting problem to find graphs that are chromatic-choosable. A natural question raised by Xuding Zhu (2013) is whether there exists a constant integer k such that G^k is chromatic-choosable for every graph G.

  Motivated by the List Total Coloring Conjecture, Kostochka and Woodall 92001) asked whether G^2 is chromatic-choosable for every graph G. Kim and Park (2013) answered the Kostochka and Woodall's question in the negative by finding a graph G such that G^2 is a complete multipartite graph. In this talk, we answer Zhu's question by showing that for every integer $k \geq 2$, there exists a graph G such that G^k is not chromatic-choosable. Moreover, for any fixed k we show that the value $\chi_l(G^k) - \chi(G^k)$ can be arbitrarily large. This is a joint work with Seog-Jin Kim and Young Soo Kwon.

- **Jongyook Park** (USTC)

  Title: *Bounds on eigenvalues for distance-regular graphs.*

  Abstract: In 1986, Terwilliger gave a result on the relationship between the eigenvalues of a distance-regular graph Γ with those of a local subgraph.

  In this talk, I will generalize this theorem to other subgraphs of Γ and give some applications of this generalization. This is joint work with Jack Koolen.

- **Vincent Pilaud** (École Polytechnique)

  Title: *Signed tree associahedra*

  Abstract: An associahedron is a polytope whose vertices correspond to the triangulations of a convex polygon and whose edges correspond to flips between them. Loday gave a particularly elegant realization of the associahedron which has been generalized in two directions: on the one hand by Hohlweg and Lange to obtain multiple realizations of the associahedron parametrized by a sequence of signs, and on the other hand by Postnikov to obtain a realization of the graph associahedra of Carr and Devadoss. The goal of this talk is to unify and extend these two constructions to signed tree associahedra. We will also present the rich combinatorial and geometric properties of the resulting polytopes. The talk will be illustrated by the case of the classical associahedron, whose interpretation in terms of spines (arXiv:1307.4391, joint work with Carsten Lange) was motivating this work. The talk is based on arXiv:1309.5222.

- **Michel Pocchiola** (Institut de Mathematiques de Jussieu)

  Title: *Arrangements of double pseudolines.*

  Abstract: An arrangement of double pseudolines is a finite family of separating simple closed curves of the projective plane that intersect pairwise in four transversal intersection points and induce pairwise a cellular decomposition of the projective plane. My presentation will focus on the combinatorics of this class of arrangements related to the algorithmics of visibility graphs.

- **Yoshio Sano** (University of Tsukuba)

  Title: *Travel groupoids on graphs.*

  Abstract: A *groupoid* is the pair $(V, *)$ of a nonempty set V and a binary operation $*$ on V. The notion of travel groupoids was introduced by L. Nebeský ("Travel groupoids", *Czechoslovak Mathematical Journal* **56(131)** (2006) 659–675) in connection with his study on geodetic graphs and signpost systems. First, let us recall the definition of travel groupoids.

  A *travel groupoid* is a groupoid $(V, *)$ satisfying the following axioms (t1) and (t2):

  (t1) $(u * v) * u = u$ (for all $u, v \in V$),

  (t2) if $(u * v) * v = u$, then $u = v$ (for all $u, v \in V$).

A *geodetic graph* is a connected graph in which there exists a unique shortest path between any two vertices. Let G be a geodetic graph, and let $V := V(G)$. For two vertices u and v of G, let $A_G(u, v)$ denote the vertex adjacent to u which is on the unique shortest path from u to v in G. Define a binary operation $*$ on V as follows: For all $u, v \in V$, let $u * v := A_G(u, v)$ if $u \neq v$ and $u * v := u$ if $u = v$. This groupoid $(V, *)$ is called the *proper groupoid* of the geodetic graph G. Remark that the proper groupoid of any geodetic graph is a travel groupoid.

Let $(V, *)$ be a travel groupoid, and let G be a graph. We say that $(V, *)$ *is on* G or that G *has* $(V, *)$ if $V(G) = V$ and $E(G) = \{\{u, v\} \mid u, v \in V, u \neq v, \text{ and } u * v = v\}$. Note that every travel groupoid is on exactly one graph.

In this talk, we first review some results by Nebeský, and then we consider the following problem: *Given a finite graph, how many travel groupoids are there on it?* We give the exact value of the number of travel groupoids on a given finite graph by using a combinatorial structure in terms of the given graph.

(This is joint work with Jung Rae CHO and Jeongmi PARK.)

- **Ferenc Szöllősi** (Tohoku University)

  Title: *Nonexistence results for complex equiangular tight frames.*

  Abstract: A finite complex equiangular tight frame (ETF) is a collection of n complex unit vectors $\varphi_1, \varphi_2, \ldots, \varphi_n$ in $\mathbb{C}^m$ having mutual inner product (or "angle" between distinct vectors φ_i and φ_j) as small as possible in absolute value. In particular, for complex ETFs the Welch bound is attained and one has

$$|\langle \varphi_i, \varphi_j \rangle| = \sqrt{\frac{n-m}{m(n-1)}} \qquad \text{for } 1 \leq i < j \leq n.$$

  We study the Gram matrices of complex equiangular tight frames and describe some new algebraic features of theirs which subsequently lead on the one hand to the nonexistence of several low dimensional complex ETFs; and on the other hand to the full algebraic classification of all complex ETFs in $\mathbb{C}^3$. We use computer aided methods, in particular, Gröbner basis calculations to conclude our results.