Abstracts of 2007 Combinatorics Workshop

Combinatorics Workshop

August 6-8, 2007

Combinatorics Lab., KAIST

Daejeon, Korea

주관 / KAIST 수리과학과 조합수학연구실

후원 / KAIST BK21 수학인재양성사업단

Abstracts of 2007 Combinatorics Workshop

Combinatorics Lab., KAIST

August 6-8, 2007

Contents

일정표

시간	8월 6일(월)		8월 7일(화)		8월 8일(수)	
	좌장	연사	좌장	연사	좌장	연사
09:30 - 10:20			7A 세션	7A-1 초청 강연 (권재훈)	8A 세션	8A-1 초청 강연 (남덕우)
10:20 - 10:40				긴 휴식		긴 휴식
10:40 - 11:10				7A-2 강연 (주형관)		8A-2 강연 (송익호)
11:10 - 11:20				휴식		휴식
11:20 - 11:50				7A-3 강연 (Jack Koolen)		8A-3 강연 (신희성)
11:50 - 12:30			점심			
12:30 - 13:00	등록		자유 토론 및 토의			
13:00 - 13:30	개회					
13:30 - 14:20	6A 세션	6A-1 초청 강연 (엄상일)	7B 세션	7B-1 초청 강연 (엄상일)	8B 세션	8B-1 초청 강연 (엄상일)
14:20 - 14:40		긴 휴식		긴 휴식		긴 휴식
14:40 - 15:30		6A-2 초청 강연 (곽진호)		7B-2 초청 강연 (강순이)		8B-2 강연 (서승현)
15:30 - 15:50		긴 휴식		긴 휴식		긴 휴식
15:50 - 16:20		6A-3 강연 (권영수)		7B-3 강연 (안지현)		8B-3 강연 (사노 요시오)
16:20 - 16:30		휴식		휴식		휴식
16:30 - 17:00		6A-4 강연 (김장수)		7B-4 강연 (신희성)		8B-4 강연 (김장수)
17:00 - 17:30	광고		광고		폐회	
17:30 - 20:00	숙소배정		연회			

6A-1 Circle Graphs Obstructions under Pivoting

Jim Geelen (University of Waterloo), 엄상일* (University of Waterloo)

Abstract

A *circle graph* is the intersection graph of a set of chords of a circle; given a set of chords of a circle, chords represent vertices, and two vertices (chords) are adjacent if and only if they intersect.

An edge uv of a graph partitions neighbors of u or v into three sets, say X_0, X_1, X_2, based on the adjacencies to u and v; *pivoting* uv is an operation to "flip" edges between X_i and X_j for distinct i, j. A *pivot-minor* of a graph G is a graph obtained by a sequence of pivoting and vertex-deletions. It is not too hard to see that every pivot-minor of a circle graph is a circle graph.

We determine the pivot-minor-minimal non-circle-graphs; there 15 obstructions. These obstructions are found, by computer search, as a corollary to Bouchet's characterization of circle graphs under local complementation. Our characterization implies Kuratowski's Theorem; a graph is planar if and only if it has no minor isomorphic to K_5 or $K_{3,3}$.

6A-2 Counting side-pairings of a polygon and Harer-Zagier Theorem

곽진호 (POSTECH)

Abstract

How many side-pairings are there of an even-gonal (rooted) polygon? We will discuss how to solve this problem and also its related problems, in particular an embedding problem of a bouquet of circles into an orientable surfaces. A related generating function can be obtained in a closed form, by Harer-Zagier Theorem. But it needs a Gaussian integral over a space of hermitian matrices. To avoid this integral method, we finally discuss a possibility of finding another combinatorial proof of Harer-Zagier Theorem.

6A-3 Reflexibility of regular Cayley maps for abelian groups

Marston Conder(Auckland University, New Zealand), 권영수* (영남대학교), Josef Siran(Open University, UK)

Abstract

In this paper, properties of reflexible Cayley maps for abelian groups are investigated, and as a result, it is shown that a regular Cayley map of valency greater than 2 for a cyclic group is reflexible if and only if it is anti-balanced.

6A-4 The largest power of 2 in the number of involutions

김동수 (KAIST), 김장수* (KAIST)

Abstract

Kummer[5] showed that the largest power of a prime p in a binomial coefficient $\binom{n}{k}$ is the number of carries in adding $n-k$ and k in radix p. With this theorem Goetgheluck[3] gave a method to calculate binomial coefficients. Alter[1] found the largest power of a prime p in the n-th Catalan number $C_n = \frac{1}{n+1}\binom{2n}{n}$ and there were some generalisations of this[4, 6].

We want to do the same thing for t_n, the number of involutions. If we factorize t_n for $n = 1, 2, \ldots, 10$ then we get

$$1, 2, 2^2, 2 \cdot 5, 2 \cdot 13, 2^2 \cdot 19, 2^3 \cdot 29, 2^2 \cdot 191, 2^2 \cdot 131, 2^3 \cdot 1187.$$

The odd primes in t_n are difficult to figure out. But it seems to have a simple formula for the power of 2 in t_n.

For an integer n a prime p, let $\xi_p(n)$ denote the largest integer k such that p^k divides n. Since we always consider $p = 2$, we will write $\xi(n) = \xi_2(n)$.

In this talk we show that

$$\xi(t_n) = \left\lfloor \frac{n}{2} \right\rfloor - 2\left\lfloor \frac{n}{4} \right\rfloor + \left\lfloor \frac{n+1}{4} \right\rfloor. \tag{1}$$

We also show that for $s \geq 3$, the smallest period in the sequence $\{\beta_n \mod 2^s\}$ is 2^{s+1}, where β_n is the odd factor of t_n. Let t_n^e and t_n^o be the number of even and odd involutions in $\S_n$ respectively. The results of this talk are shown in the following table. Here $o(n)$(resp. $e(n)$) is 1 if n is odd(resp. even) and 0 otherwise, and X_1 and X_2 are unknown functions(we conjecture for X_2 in the last part of this talk).

n	$\xi(t_n)$	$\xi(t_n^e - t_n^o)$	$\xi(t_n^e)$	$\xi(t_n^o)$
$4k$	k	k	$k + o(k)$	$k + X_1(k)$
$4k+1$	k	k	$k + X_2(k)$	$k + \xi(k) + e(k)$
$4k+2$	$k+1$	$k+3+\xi(k)$	k	k
$4k+3$	$k+2$	$k+1$	k	k

References

[1] Ronald Alter and K. K. Kubota. Prime and prime power divisibility of Catalan numbers. *J. Combinatorial Theory Ser. A*, 15:243–256, 1973.

[2] C. D. Godsil. *Algebraic combinatorics.* Chapman and Hall Mathematics Series. Chapman & Hall, New York, 1993.

[3] P. Goetgheluck. Notes: Computing Binomial Coefficients. *Amer. Math. Monthly*, 94(4):360–365, 1987.

[4] Matjaž Konvalinka. Divisibility of generalized catalan numbers. *J. Combinatorial Theory Ser. A*, doi:10.1016/j.jcta.2006.11.003, 2007.

[5] E.E. Kummer. Über die ergänzungssätze zu den allgemeinen reciprocitätsgesetzen. *J. Reine Angew. Math*, 44:93–146, 1852.

[6] Alexander Postnikov and Bruce E. Sagan. What power of two divides a weighted catalan number? *J. Combinatorial Theory Ser. A*, 114:970–977, 2007.

7A-1 Howe duality and Young tableaux

권재훈 (서울시립대학교)

Abstract

The Lie superalgebras and their representations appear naturally as the fundamental algebraic structures in many areas of mathematics and mathematical physics, and they have been studied by many people since the fundamental work of Kac [4]. Recently, from the viewpoint of Howe duality [3], a close relation between the representations of Lie algebras and Lie superalgebras has been observed (see for example, [2] and other references therein).

Let $\mathfrak{F}^n$ ($n \geq 1$) be the infinite dimensional Fock space generated by n pairs of free fermions and n pairs of free bosons (see [1, 2]). On $\mathfrak{F}^n$, there exists a natural commuting actions of the infinite dimensional Lie superalgebra $\widehat{\mathfrak{gl}}_{\infty|\infty}$ and the finite dimensional Lie algebra $\mathfrak{gl}_n$. Using Howe duality, Cheng and Wang derived a multiplicity-free decomposition

$$\mathfrak{F}^n \simeq \bigoplus_{\lambda \in \mathbb{Z}_+^n} L_\lambda \otimes L_n(\lambda),$$

as a $\widehat{\mathfrak{gl}}_{\infty|\infty} \oplus \mathfrak{gl}_n$-module, where the sum ranges over all generalized partitions λ of length n, $L_n(\lambda)$ is the rational representation of $\mathfrak{gl}_n$ corresponding to λ, and L_λ is the associated quasi-finite irreducible highest weight representation of $\widehat{\mathfrak{gl}}_{\infty|\infty}$ [2].

Suppose that $\mathcal{A}$ and $\mathcal{B}$ are linearly ordered $\mathbb{Z}_2$-graded sets. Motivated by the character formula of L_λ in [1], we introduce a new combinatorial object called *$\mathcal{A}/\mathcal{B}$-semistandard tableaux of shape* λ. Roughly speaking, an $\mathcal{A}/\mathcal{B}$-semistandard tableau of shape λ is a pair of tableaux (T^+, T^-) such that T^+ (resp. T^-) is a semistandard tableau with letters in $\mathcal{A}$ (resp. $\mathcal{B}$), where the shapes of T^+ and T^- are not necessarily fixed ones but satisfy certain conditions determined by λ. We develop the insertion scheme for $\mathcal{A}/\mathcal{B}$-semistandard tableaux, and derive analogues of Robinson-Schensted-Knuth (or simply RSK) correspondence and Littlewood-Richardson (or simply LR) rule. Then, we show that the character of $SST_{\mathcal{A}/\mathcal{B}}(\lambda)$, the set of all $\mathcal{A}/\mathcal{B}$-semistandard tableaux of shape λ, is equal to the character of L_λ with suitable choices of $\mathcal{A}$ and $\mathcal{B}$. As corollaries, we immediately obtain new combinatorial interpretations of the decomposition of the Fock space representation $\mathfrak{F}^n$ and the tensor product $L_\lambda \otimes L_\mu$ from RSK correspondence and LR rule, respectively.

References

[1] S.-J. Cheng, N. Lam, *Infinite-dimensional Lie superalgebras and hook Schur functions*, Comm. Math. Phys. **238** (2003), no. 1-2, 95–118.

[2] S.-J. Cheng, W. Wang, *Lie subalgebras of differential operators on the super circle*, Publ. Res. Inst. Math. Sci. **39** (2003), no. 3, 545–600.

[3] R. Howe, *Remarks on classical invariant theory*, Trans. Amer. Math. Soc. **313** (1989), no. 2, 539–570.

[4] V. G. Kac, *Lie superalgebras*, Adv. in Math. **26** (1977) 8-96.

[5] J.-H. Kwon, *Rational semistandard tableaux and character formula for the Lie superalgebra* $\widehat{\mathfrak{gl}}_{\infty|\infty}$, arXiv:math.RT/0605005.

7A-2 On the enumeration of certain weighted graphs

Miklos Bona(Univ. of Florida), 주형관* (전남대학교), Ruriko Yoshida (Univ. of Kentucky)

Abstract

We enumerate the weighted simple graphs with a natural upper bound condition on the sum of the weight of adjacent vertices. We also compute the generating function of the numbers of these graphs, and prove that it is a rational function. In particular, we show that the generating function for connected bipartite simple graphs is of the form $\frac{p_1(x)}{(1-x)^{m+1}}$. For nonbipartite simple graphs, we get a generating function of the form $\frac{p_2(x)}{(1-x)^{m+1}(1+x)^l}$. Here m is the number of vertices of the graph, $p_1(x)$ is a symmetric polynomial of degree at most m, $p_2(x)$ is a polynomial of degree at most $m+l$, and l is a nonnegative integer. In addition, we give computational results for various graphs.

7A-3 On a spectral characterization of the Hamming graphs

Jack Koolen (POSTECH)

Abstract

In this talk I will study the spectrum of a graph, that is the spectrum of its adjacency matrix. If two graphs G and H have the same spectrum then we say that G is cospectral with H. It was shown by Hoffman that almost all trees have a cospectral mate, that is a graph cospectral with the tree but not isomorphic. But for graphs in general it is believed that usually the graph is characterized by its adjacency matrix.

In the class of distance-regular graphs there are only a few examples known which are characterized by its spectrum. In this talk I will show that the Hamming graphs $H(3, q)$ with $q > 60$ are such graphs. This is work in progress and is joint work with Edwin van Dam (Tilburg) and Sejeong Bang (BUSAN).

7B-1 Excluding a Bipartite Circle Graph from Line Graphs

엄상일 (University of Waterloo)

Abstract

Robertson and Seymour (1991) proved that every graph of sufficiently large tree-width must contain a minor isomorphic to a fixed planar graph. Their theorem was generalized to a theorem on representable matroids by Geelen, Gerards, and Whittle (2007) stating that every matroid representable over a fixed finite field of sufficiently large branch-width must contain a minor isomorphic to a fixed planar matroid. (A *planar matroid* is a cycle matroid of a planar graph.)

We aim to prove the following conjecture, that is another generalization of Robertson and Seymour's grid theorem. Rank-width is a graph width parameter, like tree-width, introduced by Oum and Seymour (2006) to investigate clique-width. Pivot-minors of a graph G are graphs obtained from G by repeatedly applying certain operations, like minors. A *circle graph* is the intersection graph of chords in a circle. Bipartite circle graphs are related to planar graphs, shown by de Fraysseix (1981).

Conjecture: Let H be a bipartite circle graph. Every graph G with sufficiently large rank-width must have a pivot-minor isomorphic to H.

We prove that for fixed bipartite circle graph H, all line graphs with sufficiently large rank-width (or clique-width) must contain an isomorphic copy of H as a pivot-minor. To prove this, we introduce graphic delta-matroids. Graphic delta-matroids are minors of delta-matroids of line graphs and they generalize graphic or cographic matroids.

7B-2 On refinements of Ramanujan's partition congruence modulo 5

강순이 (고등과학원)

Abstract

A partition λ of a positive integer n is a weakly decreasing sequence of positive integers $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_k$ such that $\lambda_1+\lambda_2+\cdots+\lambda_k = n$ and a partition function $p(n)$ counts the number of partitions of n. Ramanujan's discovery of partition congruences $p(5n+4) \equiv 0 \pmod 5$, $p(7n+5) \equiv 0 \pmod 7$, and $p(11n+6) \equiv 0 \pmod{11}$ inspired tremendous work in the theory of partition from both analytical and combinatorial points of view. Dyson was the first to consider combinatorial explanations of these congruences and defined the rank of a partition as the largest part minus the number of parts and made the empirical observations that the residue of the rank mod 5 (resp. mod 7) divides the partitions of $5n+4$ (resp. $7n+5$) into five (resp. seven) equal classes. As his rank failed to explain the last congruence, Dyson conjectured the existence of a hypothetical statistic, called the crank, for the congruence mod 11. Many conjectures by Dyson on the rank was proved by Atkin and Swinnerton-Dyer and a crank for all three Ramanujan congruences was found by Andrews and Garvan. Their crank is defined as the largest part if there is no 1 and as the number of parts which are larger than the number of ones, otherwise. Later, Garvan, Kim, and Stanton found different cranks, which also explained all three congruences. Their approach made essential use of t-cores of partitions and led to explicit bijections between various equinumerous classes. On the other hand, in a study of sign-balanced labelled posets, Stanley introduced a new partition statistic $srank(\lambda) = \mathcal{O}(\lambda) - \mathcal{O}(\lambda')$, where $\mathcal{O}(\lambda)$ denotes the number of odd parts of the partition λ and λ' is the conjugate of λ. Using an analytic method, Andrews found a refinement of Ramanujan's partition congruence mod 5: $p_0(5n+4) \equiv p_2(5n+4) \equiv 0 \pmod 4$, while $p(n) = p_0(n) + p_2(n)$. Here, $p_i(n)$ $(i = 0, 2)$ denotes the number of partitions of n with $srank \equiv i \pmod 4$. Recently, Berkovich and Garvan found three partition statistics, the ST-crank, the 2-quotient-rank and the Garvan-Kim-Stanton 5-core-crank, that divide the partitions enumerated by $p_i(5n+4)$ $(i = 0, 2)$ into five equinumerous classes. Moreover, they discovered another new partition statistic, named BG-rank, which gives a more general refinement than Andrews refinement of Ramanujan's partitions congruence modulo 5. But there is a different refinement of the same congruence. Let $M(k, m, n)$ be the number of partitions of n with Andrews-Garvan crank congruent to k modulo m. A few years ago, Garvan found a refinement of Ramanujan's congruence mod 5, $M(k, 2, 5n+4) \equiv 0 \pmod 5$, $k = 0, 1$ together with the combinatorial interpretation $M(2k+\alpha, 10, 5n+4) = \frac{M(\alpha,2,5n+4)}{5}$, $0 \leq k \leq 4$, with $\alpha = 0, 1$. In this talk, we survey on these refinements of Ramanujan's partition congruence modulo 5 and various partition statistics and present a new proof of Garvan's refinement and interrelations among the refinements.

7B-3 Certain Ranked Posets of Length One and Continued Fractions

안지현* (전남대학교), 주형관 (전남대학교)

Abstract

We enumerate certain ranked posets of length one as compute corresponding generating function. Diamond graph is associated poset and it's recursion formula is continued fraction.

References

[1] M. Aigner, *Combinatorial Theory*, Springer, Berlin, 1979.

[2] M. Bona, H.-K. Ju and Ruriko Yoshida, *On the Enumeration of Certain Weighted Graphs*, Submitted at Discrete Applied Math., Preprint available at http://front.math.ucdavis.edu/math.CO/0606163.

[3] M. Bona, H.-K. Ju and Ruriko Yoshida, *Zigzag Posets and Wall Posets*, In Preparation.

[4] J. Farley, *Linear Extensions of Ranked Posets, Enumerated by Descents. A Problem of Stanley from the 1981Banff Conference on Ordered Sets*, Advances in Applied Math., 34(2005), 295-312.

[5] R. Stanley, *Ordered Structures and Partitions*, Mem. Amer. Math. Soc., vol. 119, American Mathematical Society, Providence, RI, 1972.

[6] R. Stanley, *Two Poset Polytopes,* Discrete Comput. Geom.,**1** (1986), 9-23.

[7] R. Stanley, *Enumerative Combinatorics*, Vol.I., Cambridge Studies in Advanced Mathematics 49, Cambridge University Press, 1997.

7B-4 On a decomposition of simple polytopes

최수영 (KAIST), 신희성* (KAIST)

Abstract

A convex d-polytope is called *simple* if exactly d facets of P meet at each vertex. A simple polytope P is *reducible* if P can be decomposed into a Cartesian product of polytopes of lower dimensions. In this paper, we show that P can be decomposed into simplices if P has triangular or cubical 2-faces. Moreover, we find that necessary and sufficent condition that P is equivalent to a product of simplices and polygons, essentially, the information of 2-faces determines the full structure of a simple polytope for certain cases. Finally, we make the criterion to decide the reduciblity of the simple polytope.

8A-1 Modeling and analyses for biological networks

남덕우* (국가수리과학연구소), 서승현 (청주대학교), 김상수 (숭실대학교)

Abstract

Biological processes are carried out by the complex activities of genes, proteins and metabolites, which are often represented by graph models. We introduce key biological networks such as gene regulatory networks and protein-protein interaction networks, and discuss their their topological properties. We consider two kinds of topics for mathematical scientists: reconstructing biological networks from given experimental data under a mathematical model, and analyzing the graph structures of known biological networks. For the first topic, we deal with how to reconstruct gene regulatory networks from a large scale gene expression data. We especially focus on the Boolean network model. We discuss fast network search algorithms and related combinatorial problems. We show how a randomized algorithm can improve the search speed. For the second topic, we discuss how the biological networks are distinguished from randomly connected networks by frequently observed local patterns. We review the literature on this topic.

8A-2 Polynomial representation for the number of partitions with length fixed

송익호* (KAIST), 박소령 (가톨릭대학교), 강현구 (KAIST), 배진수 (세종대학교)

Abstract

Let $N(n)$ and $M(n,k)$ denote the number of partitions of n and that with length k (or with k parts), respectively, where n and k are nonnegative integers. Obviously,

$$M(n,k) \quad = \quad 0, \quad \text{if } k > n. \tag{2}$$

Defining

$$N(0) \quad = \quad 1 \tag{3}$$

and

$$M(0,0) \quad = \quad 1 \tag{4}$$

for convenience, we have

$$N(n) \quad = \quad \sum_{k=0}^{n} M(n,k). \tag{5}$$

It is well-known that the number $M(n,k)$ satisfies the recursion

$$M(n,k) \quad = \quad M(n-1,k-1) + M(n-k,k), \tag{6}$$

with which we can evaluate the number $M(n,k)$ recursively once n and k are given, using $M(n,0) = 0$ for $n \geq 1$ and $M(0,0) = 1$.

In this paper, we are interested in finding a non-recursive representation for the evaluation of $M(n,k)$. We will first show that the number $M(n,k)$ can be described by a set of $\widetilde{k}$ polynomials of degree $k-1$ in $Q_{n,\widetilde{k}}$, where $\widetilde{k}$ denotes the least common multiple of the k integers $1, 2, \ldots, k$ and $Q_{n,\widetilde{k}}$ denotes the quotient of n when divided by $\widetilde{k}$. In addition, the sets of the $\widetilde{k}$ polynomials are obtained and shown explicitly for $k = 3, 4, 5$, and 6. For example, denoting by $R_{n,m}$ the quotient of n when divided by m, we have

$$M(n,3) \quad = \quad 3Q_{n,6}^2 + R_{n,6}Q_{n,6} + c_{0,3}(R_{n,6}), \tag{7}$$

where $\{c_{0,3}(r)\}_{r=0}^{5} = \{0,0,0,1,1,2\}$, and

$$\begin{aligned} M(n,4) \quad = \quad & 12Q_{n,12}^3 + 3(R_{n,12}+1)Q_{n,12}^2 \\ & + c_{1,4}(R_{n,12})Q_{n,12} + c_{0,4}(R_{n,12}), \end{aligned} \tag{8}$$

where

$$c_{1,4}(r) \quad = \quad \begin{cases} (r^2+2r)/4, & R_{r,2} = 0, \\ (r^2+2r-3)/4, & R_{r,2} = 1, \end{cases} \tag{9}$$

and $\{c_{0,4}(r)\}_{r=0}^{11} = \{0,0,0,0,1,1,2,3,5,6,9,11\}$.

Presented at the CMS-MITACS Joint Conference, Winnipeg, MB, Canada, June 2007; Accepted for publication in *Mathematics of Computation.*

8A-3 Maple 소개 및 기초

신희성 (KAIST)

Abstract

Maple을 소개하고 그 사용법에 대해서 발표하겠습니다.

1. Maple 소개
2. Maple 설치
3. Maple 동작 방법
4. 기초적인 Maple 명령어
5. Maple 이용 예제

8B-1 Rank-width and Well-quasi-ordering

엄상일 (University of Waterloo)

Abstract

Robertson and Seymour (1990) proved that graphs of bounded tree-width are well-quasi-ordered by the graph minor relation. By extending their arguments, Geelen, Gerards, and Whittle (2002) proved that binary matroids of bounded branch-width are well-quasi-ordered by the matroid minor relation. We prove another theorem of this kind in terms of *rank-width* and *vertex-minors*. For a graph $G = (V, E)$ and a vertex v of G, a *local complementation* at v is an operation that replaces the graph induced on neighbors of v by its complement graph. A graph H is called a *vertex-minor* of G if H can be obtained by applying a sequence of vertex-deletions and local complementations. Rank-width was defined by Oum and Seymour to investigate clique-width; they showed that graphs have bounded rank-width if and only if they have bounded clique-width. We prove that graphs of bounded rank-width are well-quasi-ordered by the vertex-minor relation; in other words, for every infinite sequence $G_1, G_2, \ldots$ of graphs of rank-width (or clique-width) at most k, there exist $i < j$ such that G_i is isomorphic to a vertex-minor of G_j. This implies that there is a finite list of graphs such that a graph has rank-width at most k if and only if it contains no one in the list as a vertex-minor. The proof uses the notion of *isotropic systems* defined by Bouchet.

8B-2 Threshold Arrangements

서승현 (청주대학교)

Abstract

A hyperplane arrangement $\mathcal{A}$ is a collection of finitely many affine hyperplanes H in a vector space V. (Most often we will take $V = \mathbb{R}^n$.) Their study lies at the intersection of combinatorics, topology, geometry and Lie algebra, and most questions about them red+K4uce to combinatorics. The simplest invariants of $\mathcal{A}$ is its number of regions $r(\mathcal{A})$, i.e., the number of connected components of the set $\mathbb{R}^n - \cup_{H\in\mathcal{A}} H$. Note that $r(\mathcal{A})$ equals the dimension of the cohomology ring of the complement to the complexification of $\mathcal{A}$. Let $\mathcal{R}(\mathcal{A})$ denote the set of regions of $\mathcal{A}$.

Now we consider a special hyperplane arrangement. The *Threshold arrangement* $\mathcal{T}_n$ is given by the hyperplanes $x_i + x_j = 0$, for $1 \le i < j \le n$. The exponential generating function for the number of regions of $\mathcal{T}_n$ is given by

$$\sum_{n\ge 0} r(\mathcal{T}_n)\frac{x^n}{n!} = \frac{e^x(1-x)}{2-e^x}. \tag{10}$$

Also the egf for the characteristic polynomial $\chi_{\mathcal{T}_n}(t)$ of $\mathcal{T}_n$ is given by

$$\sum_{n\ge 0} \chi_{\mathcal{T}_n}(t)\frac{x^n}{n!} = (1+x)(2e^x-1)^{(t-1)/2}, \tag{11}$$

where $\chi_{\mathcal{A}}(t)$ is defined by the rank polynomial of the intersection poset for $\mathcal{A}$. There are some bijective proofs of (10) by giving a bijection from $\mathcal{R}(\mathcal{T}_n)$ to the set of threshold graphs on n nodes. But (11) has been proved with finite field methods only. In particular, combinatorial interpretation of coefficients of $\chi_{\mathcal{T}_n}(t)$ as the number of threshold graphs is still open.

In this talk, we discuss refined enumeration of threshold graphs, which explain coefficients of $\chi_{\mathcal{T}_n}(t)$ combinatorially. Also we present the number of regions and the characteristic polynomial of "Shi threshold arrangements given by the hyperplanes $x_i + x_j = 0\,, 1$, for $1 \le i < j \le n$. Other variations of threshold arrangements will be mentioned, too.

8B-3 The greedy algorithm for strict cg-matroids

사노 요시오 (POSTECH)

Abstract

A matroid which was introduced by H. Whitney in 1935 is one of the most important structures in combinatorial optimization. Many researchers have studied and extended the matroid theory. One of the reasons that matroids are important is that matroids are closely related to the greedy algorithm, which solves the maximum base problem efficiently. A matroid-like structure defined on a convex geometry, called a cg-matroid, is defined by S. Fujishige, G. A. Koshevoy, and Y. Sano. Strict cg-matroids are the special subclass of cg-matroids.

A *convex geometry* $(E, \mathcal{F})$ is the pair of a finite set E and a family $\mathcal{F}$ of subsets of E which satisfies the following: (i) $\emptyset,\ E \in \mathcal{F}$. (ii) $X, Y \in \mathcal{F} \Rightarrow X \cap Y \in \mathcal{F}$. (iii) For any $X \in \mathcal{F} \setminus \{E\}$, there exists $e \in E \setminus X$ such that $X \cup \{e\} \in \mathcal{F}$. A *cg-hereditary system* $(E, \mathcal{F}; \mathcal{I})$ is the pair of a convex geometry $(E, \mathcal{F})$ and a subfamily $\mathcal{I}$ of $\mathcal{F}$ which satisfies the following: (I0) $\emptyset \in \mathcal{I}$. (I1) $I_1 \in \mathcal{F}$, $I_2 \in \mathcal{I}$, $I_1 \subseteq I_2 \Rightarrow I_1 \in \mathcal{I}$. A cg-hereditary system $(E, \mathcal{F}; \mathcal{I})$ is called a *strict cg-matroid* if $\mathcal{I}$ satisfies the following property:

(ISA) (Strict Augmentation Property)
For any $I_1, I_2 \in \mathcal{I}$ with $|I_1| < |I_2|$,
there exists $e \in \tau(I_1 \cup I_2) \setminus I_1$ such that $I_1 \cup \{e\} \in \mathcal{I}$,

where τ is the closure operator of the convex geometry $(E, \mathcal{F})$.

In this talk, I will talk about the relation between strict cg-matroids and the greedy algorithm. Let $(E, \mathcal{F}; \mathcal{I})$ be a cg-hereditary system and $w : E \to \mathbb{R}_{\geq 0}$ be a weight function. Now we consider the following optimization problem.

(P) maximize $\Sigma_{e \in I} w(e)$, subject to $I \in \mathcal{I}$.

For this optimization problem, we will show that the greedy algorithm gives an optimal solution of the problem (P) if the cg-hereditary system $(E, \mathcal{F}; \mathcal{I})$ is strict cg-matroids and the wight function w is a natural weighting. And we also show that the greedy algorithm works for a cg-hereditary system with any natural weighting if and only if the cg-hereditary system is a strict cg-matroid.

8B-4 조합론에서의 Maple 활용

김장수 (KAIST)

Abstract

In this talk we use Maple to investigate the the following.

1. The largest power of the number of involutions
2. Robinson-Schensted algorithm and domino insertion algorithm
3. Statistics(inv, des, maj, exc) of permutations and Foata map
4. Testing unimodality and logconcavity of a sequence
5. Generating functions