

POW 2025-13: Covering the Donut with Unit Punches

수리과학과 석박통합과정 김준홍

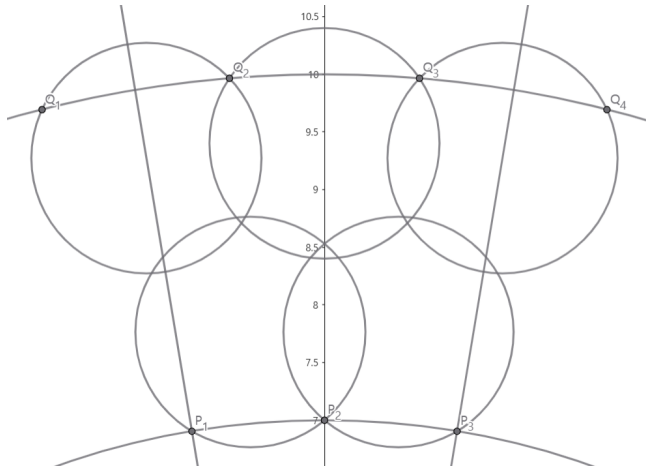


Figure 1: Cover with 76 circles

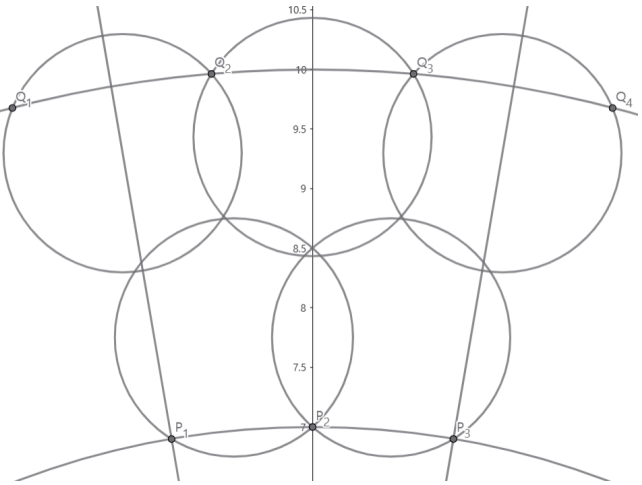


Figure 2: Failed cover with 74 circles

Let C_1, C_2 are two circles of radius 7 and 10 that have same center O .

For Figure 1, $P_1, P_2, P_3, \dots, P_{38}$ and $Q_1, Q_2, Q_3, \dots, Q_{38}$ are vertices of inscribed regular 38-gon to C_1 and C_2 , respectively.

Arrange 38-gon so that O, P_1 , and midpoint of $\overline{Q_1Q_2}$ are on the same line.

Now, draw circle that has radius 1, P_1 and P_2 are on the circle, and center is inside the annulus.

Do the same with P_2 and P_3, P_3 and P_4, \dots and so on. Then we get 38 circles.

Repeat same step to Q_1 and Q_2, Q_2 and Q_3, \dots and so on, then we get another 38 circles.

These 76 circles cover annulus, shown on the Figure 1.

(\because This covers $1/38$ of annulus completely, and arrangement is symmetric)

But we cannot do this with 74 circles, shown in Figure 2.