POW2025-13
COVERING THE DONUT WITH UNIT PUNCHES

KAIST 2024 MINKYU SHIN

Problem. Each punch can be centered anywhere in the plane and removes all points within distance 1 from its center. What is the minimum number of punches needed to remove every point in the annulus between the circles of radius 7 and 10 (with the same center)? Describe your construction. The person with the smallest number of punches earns +4, and the next four best answers earn +3.

Solution. There is a covering with 76 punches.

Let

$$r = 7.75,$$
$$s = \sqrt[4]{7817} \approx 9.403,$$
$$\theta = \tan^{-1}\left(\frac{\sqrt{202\sqrt{7817} - 17618}}{99 + \sqrt{7817}}\right) \approx 0.0827 \text{ (rad)}.$$

We consider the following disks:

$$C_k : (x - s\cos(2(k-1)\theta))^2 + (y - s\sin(2(k-1)\theta))^2 \leq 1$$

and

$$D_k : (x - r\cos((2k-1)\theta))^2 + (x - r\sin((2k-1)\theta))^2 \leq 1$$

with $k = 1, 2, ..., 38$. These 76 disks cover the given annulus. In particular, since the disks C_k and D_k with varying k are arranged with angle 2θ apart, and $2\pi/2\theta = \pi/\theta \approx 37.967$, we see that $k = 38$ is sufficient. The picture on the next page (drawn using GeoGebra geometry) illustrates a small part of the covering.

Email address: `minkyushin@kaist.ac.kr`

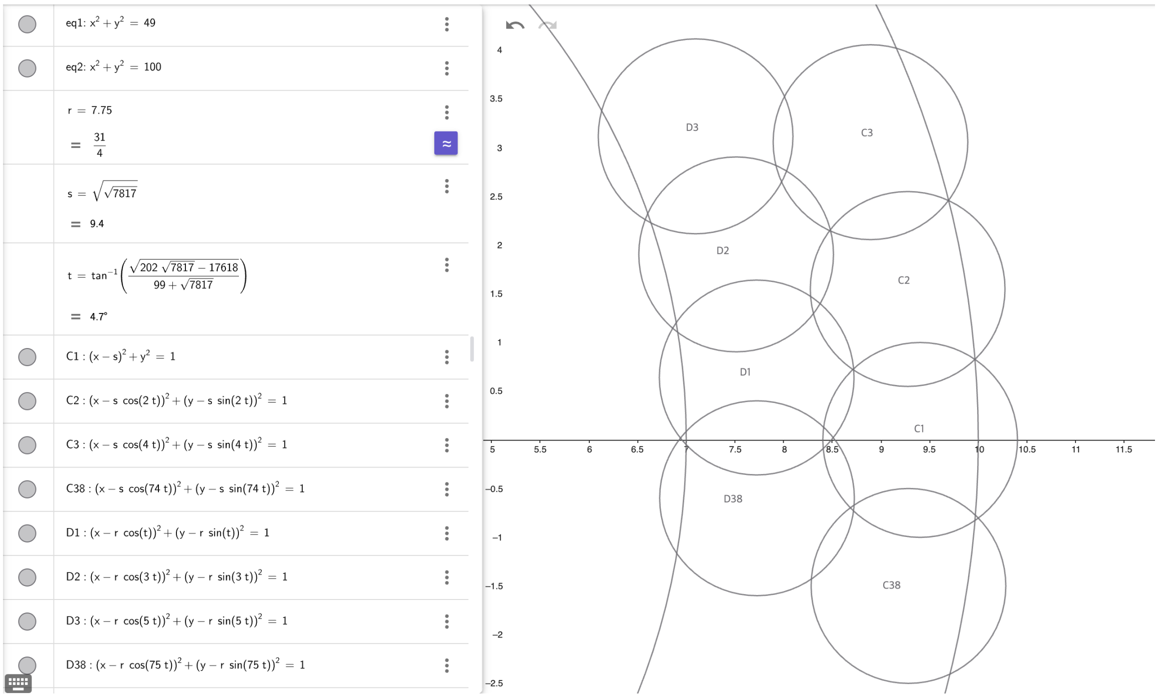

FIGURE 1. Here, θ is replaced with t.