\[
P(23 - 10)
\]

So, \( p^4 + q^4 + 1 = p^4q + p^4q + 1 = (p^4 + 1)(q^4 + 1) \pmod{19} \).

Since \( p > 1, q^4 + 1 \) and \( p, q \) are prime, \( p \mid q^4 + 1 \) and \( q \mid q^4 + 1 \).

WLOG, \( p > q \). Let \( a_q \) be the smallest number such that \( q^a + 1 \equiv 1 \pmod{p} \).

Since \( q^4 + 1 \equiv 1 \pmod{19} \), \( a_q \mid 19 \) (If not, then \( \exists \, 0 < r < a_q \) such that \( q^r \equiv 1 \pmod{19} \), which is contradiction.)

So, \( a_q = 1, 3, 9, 29 \). For the case of \( a_q = 1, 9, \) since \( \forall q \in \mathbb{Z}, \) \( a_q \) could not be \( 1, 9 \). So \( a_q \) should be \( 2, 29 \).

Suppose \( q > 2 \), then \( q^4 = (q^2)^2 \equiv (q^2)^2 \pmod{p} \), \( q^2 \equiv 1 \pmod{p} \).

Since \( p > q \), \( p = q^2 + 1 \). However, \( p, q \) are prime, so \( p \neq q + 1 \).

For \( q = 2 \), then \( 2^2 = 1 \pmod{p} \), \( p = 2147 \).

Since \( 2^2 + 5^4 + 1 = 3120 \), (2.5) satisfies the condition.

By Fermat's Little Theorem, \( q^p = 1 \pmod{p} \). By the definition of \( a_q \), \( a_q = p - 1 \), so \( p = 2a_q + 1 \) for some \( k \in \mathbb{Z} \). Then

\( p^4 + 1 = (2a_q + 1)^4 + 1 = 9, 9(9, 1) + 1 \equiv 0 \pmod{a_q} \) (for some polynomial \( G(9, k) \)).

This indicates that \( q \mid 2 \), which means \( q = 2 \).

As seen in i) \( a_q = 2 \), (2.5) satisfies the condition.

Hence, (2.5) is the only pair for \( p_q \mid p^4 + q^4 + 1 \), where \( p, q \) are prime.