1 Solution

If \( n = 1 \), then obviously the equality holds. Assume that the equality holds when the number of positive real numbers is less than \( n \). Then, for positive real numbers \( d_1, d_2, \ldots, d_n \),

\[
\sum_{\sigma \in S_n} \frac{1}{d_{\sigma(1)}(d_{\sigma(1)} + d_{\sigma(2)}) \cdots (d_{\sigma(1)} + \cdots + d_{\sigma(n)})} = \frac{1}{d_1 + \cdots + d_n} \sum_{\sigma \in S_n} \frac{d_{\sigma(1)}(d_{\sigma(1)} + d_{\sigma(2)}) \cdots (d_{\sigma(1)} + \cdots + d_{\sigma(n)})}{d_1 + \cdots + d_n}
\]

\[
= \frac{1}{d_1 + \cdots + d_n} \sum_{\sigma \in S_n} \frac{d_{\sigma(1)}(d_{\sigma(1)} + d_{\sigma(2)}) \cdots (d_{\sigma(1)} + \cdots + d_{\sigma(n-1)})}{d_1 + \cdots + d_n}
\]

\[
= \frac{1}{d_1 + \cdots + d_n} \left( \sum_{\sigma \in S_n, \sigma(n)=1} \frac{1}{d_{\sigma(1)}(d_{\sigma(1)} + d_{\sigma(2)}) \cdots (d_{\sigma(1)} + \cdots + d_{\sigma(n-1)})} \right)
\]

\[
+ \sum_{\sigma \in S_n, \sigma(n)=2} \frac{1}{d_{\sigma(1)}(d_{\sigma(1)} + d_{\sigma(2)}) \cdots (d_{\sigma(1)} + \cdots + d_{\sigma(n-1)})}
\]

\[
+ \cdots + \sum_{\sigma \in S_n, \sigma(n)=n} \frac{1}{d_{\sigma(1)}(d_{\sigma(1)} + d_{\sigma(2)}) \cdots (d_{\sigma(1)} + \cdots + d_{\sigma(n-1)})}
\]

\[
= \frac{1}{d_1 + \cdots + d_n} \left( \frac{1}{d_2d_3 \cdots d_n} + \frac{1}{d_1d_3 \cdots d_n} + \frac{1}{d_1d_2 \cdots d_{n-1}} \right)
\]

\[
= \frac{1}{d_1d_2 \cdots d_n}
\]

Therefore, by induction on \( n \), the equality in the problem holds.