Problem.
Show that for any triangle $T$ and any Jordan curve $C$ in the Euclidean plane, there exists a triangle inscribed in $C$ which is similar to $T$.

Solution.
If we fix two vertices $N, M$ of a triangle, other vertex $L$ to make triangle $\triangle NML$ similar to $T$ (with same vertices order and does not allow reflection) is unique, and $T$ is continuous subject to $N$, and $M$. For some 2-dimensional transformation matrix $R$, $T = R(N - M) + M$.

Assume we found two triangles $\triangle N_0M_0L_0$, $\triangle N_1M_1L_1$ which are similar to $T$, such that $N_0, N_1, M_0, M_1$ is on the curve $C$, $L_0$ is in the inside of $C$, and $L_1$ is in the outside of the curve $C$. We can continuously move $N_0$ and $M_0$ to $N_1$ and $M_1$, $L_0$ moves continuously to $L_1$ while crossing $C$.

To find $\triangle N_0M_0L_0$, take any point $P$ in the inside of $C$, and find the maximum possible disk centered in $C$, and do not intersect with the outside of $C$. such disk will intersect with $C$ with a point $N_0$. We can draw a triangle $\triangle N_0ML$, inscribed in the disk, which is similar to $T$. We will also assume $ML$ as the longest side of the triangle.

By rotating $\triangle N_0ML$ with center $N_0$ continuously, while either one of $M$ or $L$ will meet with $C$. (If not, trace of $M$ will form closed curve $D$, which is in the inside of $C$, since $M$ is in the inside of $C$. However $N_0$, which on the curve $C$, should be in the inside of $D$, since $N_0$ is the center of rotation, which leads to contradiction.) Without lost of generality, let’s assume $M$ met with $C$. $M$ and $L$ can be chosen as $M_0$ and $L_0$. $N_0$ and $M_0$ lies on $C$, and $L_0$ is in the inside of $C$ or lies on $C$.

To find $\triangle N_1M_1L_1$, let’s choose two furthest points of $C$ as $N_1$ and $M_1$, and choose $L_1$ such that $\triangle N_1M_1L_1$ is similar to $T$. We have set $ML$ as the longest side of the triangle, so $\overline{M_1L_1} \geq \overline{N_1M_1}$, and $L_1$ should be in the outside of $C$ or lie on $C$.

If $L_0$ or $L_1$ lies on $C$, such triangle $\triangle N_0M_0L_0$ or $\triangle N_1M_1L_1$ will complete the proof. Otherwise, we can continuously move $\triangle N_0M_0L_0$ from $\triangle N_1M_1L_1$ to find the triangle which $L$ lies on $C$. 
Technically, let’s denote parametrization of $C$ as $\varphi$. ($\varphi : [0,1] \to \mathbb{R}^2$, $\varphi$ is continuous, $\varphi(0) = \varphi(1)$, and restriction of $\varphi$ to $[0,1]$ is injective.) Let’s define $n_0, n_1, m_0, m_1$ to be reals which meets $N_0 = \varphi(n_0), N_1 = \varphi(n_1), M_0 = \varphi(m_0), M_1 = \varphi(m_1)$. For $t \in [0,1]$, we can denote move of $L$ using following parametrization: $N_t = \varphi(tn_1 + (1-t)n_0), M_t = \varphi(tm_1 + (1-t)m_0), L_t = R(N_t - M_t) + M_t$. $L_t$ is continuous subject to $N_t$ and $M_t$, also $N_t$ and $M_t$ is subject to $t$. So $L_t$ is continuous subject to $T$. This means $L_t$ is parametrization of arc connecting $L_0$ and $L_1$. If $L_0$ is in the inside of $C$, and $L_1$ is in the outside of $C$, which is well-defined by Jordan Curve Theorem, there must be $v \in (0,1)$ such that $L_v$ is on the curve $C$. (We can take supremum of $t$ such that $L_t$ is in the inside of the $C$.) $\triangle N_vM_vL_v$ is triangle which inscribed in $C$, which is also similar to $T$. \hfill \Box