Let's prove $l(n) = 2^{n-1}, u(n) = n^3 2^{n+1}$ (so that $\dfrac{u(n)}{l(n)} = 4n^3$) satisfies the condition of the problem. We may assume $n > 1$.

For any $s < 2^{n-1}$ and $b = (b_1, b_2, \cdots, b_s) \in \{0,1\}^s$ we can pick $n-1$ consecutive terms of b and make a subsequence. The number of possible subsequences of b is at most $s < 2^{n-1}$ so there is a sequence $c = (c_1, c_2, \cdots, c_{n-1}) \in \{0,1\}^{n-1}$ such that c is not a subsequence of $n-1$ consecutive terms of b. Let's define $a_{i,j}$ to be $i+1$ if $i < n$ and $j = c_i$, 1 if not. At room $i < n$ of this maze the correct guess of c_i will take you to room $i+1$ and the wrong guess will take you to room 1. So to go to room n starting at room 1 the correct guess of $c_1, c_2, \cdots, c_{n-1}$ must be made consecutively which is not possible for the sequence b. For any $s < 2^{n-1}$ and $b = (b_1, b_2, \cdots, b_s) \in \{0,1\}^s$ there is a maze such that b does not visit every room and $l(n) = 2^{n-1}$ satisfies the condition of the problem.

The total number of possible mazes (starting conditions) is at most $n^{2n} \times n = n^{2n+1}$ by determining the $a_{i,j}$ and the starting point. Let's set $s = n^3 2^{n+1}$ and consider all 2^s sequences $b = (b_1, b_2, \cdots, b_s) \in \{0, 1\}^s$. For a fixed maze and $i \in [n]$, at any starting point there is a sequence of less than n terms that visits room i so the number of sequences of length n that does not visit room i is at most $2^n - 1$. For the fixed maze and i, the number of sequences of length s that does not visit room i is at most $(2^n - 1)^{\frac{s}{n}}$ and the number of sequences of length s such that there is a maze where the sequence does not visit all the rooms is at most $n^{2n+1} \times n \times (2^n - 1)^{\frac{s}{n}} = n^{2(n+1)}(2^n - 1)^{\frac{s}{n}}$.

$$\begin{aligned}
\ln \frac{n^{2(n+1)}(2^n-1)^{\frac{s}{n}}}{2^s} &= 2(n+1)\ln n + \frac{s}{n}\ln(1 - \frac{1}{2^n}) \\
&< 2(n+1)(n-1) - \frac{s}{n}\frac{1}{2^n} = 2(n^2-1) - 2n^2 < 0
\end{aligned}$$

so $n^{2(n+1)}(2^n - 1)^{\frac{s}{n}} < 2^s$ and there is a sequence of length s which visits every room for every maze. $u(n) = n^3 2^{n+1}$ satisfies the condition of the problem.