Solution of pow2020-02

Sunghun Ko

March 2020

Let's assume there exist group G and its proper subgroups H and K such that $G = H \cup K$. If $H \subset K$, then $G = K$ thus we get a contradiction. Therefore $H \not\subset K$. Let h be the element of $H \setminus K$, and k be the arbitrary element of K. If there exists $k \in K$ such that $hk \in K$, then $h = (hk)k^{-1} \in K$ and we get contradiction. Therefore $\forall k \in K$, $hk \in H$. But then $hK \subset H$, which implies that $K = h^{-1}hK \subset h^{-1}H = H$ thus $G = H$, however this is impossible since H is a proper subgroup of G. Thus their is no such group G. $\square$