pow 2019-21's solution

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Let λ be a singular value of a A, which is not contained in the interval $(1 - \delta, 1 + \delta)$. Then λ^2 is an eigenvalue of $A^T A$. When x is an eigenvector who corresponds to λ^2 with norm of 1, $|(A^T A - I)x| = |(1 - \lambda^2)x| = |1 - \lambda^2|$. Since λ is not contained in $(1 - \delta, 1 + \delta)$, and δ is less than 1, $1 - \lambda^2 \geq 2\delta - \delta^2 > \delta$ or $\lambda^2 - 1 \geq 2\delta + \delta^2 > \delta$ hence $\left\| A^T A - I \right\| > \delta$, which contradicts to given condition. Thus there is no such λ, and every singular value of A is contained in $(1 - \delta, 1 + \delta)$.