2019-12 (Taegyun Kim)

if $p \mid 2^{m_0}+3^{n_0}$ for some $m_0, n_0 \in \mathbb{Z}$

$\Rightarrow p \mid 2^{(p-1)n^2+m_0}+3^{(p-1)n^2+n_0}$

$\therefore p \mid 2^{f(n)}+3^{f(n)}$ for some $f(m), g(n)$

$\Longleftrightarrow p \mid 2^{m_0}+3^{n_0}$ for some $m_0, n_0 \in \mathbb{Z}$

$5 \mid 2^2+3^0$, $7 \mid 2^2+3^1$, $11 \mid 2^3+3^1$

$13 \mid 2^2+3^2$ $17 \mid 2^4+3^0$, $19 \mid 2^4+3^1$

for 23, $\left(\frac{2}{23}\right)=\left(\frac{3}{23}\right)=1$. but $\left(\frac{-1}{23}\right)=-1$

$\therefore$ if $23 \mid 2^m+3^n$ for some $m, n \in \mathbb{Z}$

$\Rightarrow 3^n \equiv -2^{+m} \pmod{23}$

$\Rightarrow \left(\frac{-1}{23}\right)=1$ (contradiction)

$\therefore 23 \nmid 2^m + 3^n$ for $\forall\, m, n \in \mathbb{Z}$

$\therefore$ the smallest prime satisfying the condition is 23.