For any prime p and k such that $p^k \leq m < p^{k+1}$, we have

$$\sum_{i=1}^{\infty} \lfloor \frac{mn}{p^i} \rfloor - \sum_{i=1}^{\infty} n \lfloor \frac{m}{p^i} \rfloor - \sum_{i=1}^{\infty} \lfloor \frac{n}{p^i} \rfloor = \sum_{i=1}^{k} \left(\lfloor \frac{mn}{p^i} \rfloor - n \lfloor \frac{m}{p^i} \rfloor \right) + \sum_{i=1}^{\infty} \left(\lfloor \frac{\frac{m}{p^k} n}{p^i} \rfloor - \lfloor \frac{n}{p^i} \rfloor \right) \geq 0.$$

This proves $(m!)^n (n!)$ divides $(mn)!$. $\square$