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WLOG assume $a_1 < \cdots < a_m$. And suppose that least common multiple of a_i, a_j is less or equal to N for $1 \leq i, j \leq m$.
For $i \geq 1$, (a_i, a_{i+1}) divides $(a_{i+1} - a_i)$.

$$N \geq l.c.m\,(a_{i+1}, a_i) = \frac{a_i a_{i+1}}{(a_i, a_{i+1})} \geq \frac{a_i a_{i+1}}{a_{i+1} - a_i} \Rightarrow \frac{1}{a_i} - \frac{1}{a_{i+1}} \geq \frac{1}{N} \tag{1}$$

Since $a_i \geq i$

$$\frac{1}{\left[\frac{m}{2}\right]} > \frac{1}{a_{[m/2]}} - \frac{1}{a_m} \geq \frac{m - \left[\frac{m}{2}\right]}{N} \Rightarrow N > \left[\frac{m}{2}\right]\left(m - \left[\frac{m}{2}\right]\right) = \left[\frac{m^2}{4}\right] \geq \left[\frac{4N}{4}\right] = N \tag{2}$$

So contradiction.