2017 FALL PROBLEM OF THE WEEK
POW2017-22

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Problem. *Let p, q, r be positive integers such that $p, q \geq r$. Ada and Betty independently read all source codes of their programming project. Ada found p bugs and Betty found q bugs, including r bugs that Ada found. What is the expected number of remaining bugs that neither Ada nor Betty found?*

SOLUTION. Let N be the total number of bugs. We shall estimate N. Suppose that each bug was independently found by Ada and Betty with probability p_1 and p_2, respectively. On the one hand, because Ada found p bugs among N total bugs, it is reasonable to say that p_1 is approximately equal to $\frac{p}{N}$. On the other hand, since Ada found r bugs among q bugs that Betty found and Ada's work was independent from Betty's, it is also feasible to claim that p_1 is approximately equal to $\frac{r}{q}$. Thus, by examining Ada's work we see that $\frac{p}{N}$ is approximately equal to $\frac{r}{q}$, which implies that the expected number of N is $\frac{pq}{r}$ (this is acceptable as p, q, r are positive integers). By examining Betty's work with p_2 we derive the same estimation for N, due to symmetry. Hence the expected number of remaining bugs that neither Ada nor Betty found is

$$\frac{pq}{r} - p - q + r = \frac{(p-r)(q-r)}{r}. \qquad \square$$