POW21

2015. 국윤범

Define $f_n(x) := \begin{cases} (n!)\,x^n, & |x| \le \frac{1}{2^n} \\ 0 & \text{o.w.} \end{cases}$ for all $n \in \mathbb{N}$ and $f(x) := \sum_{n=0}^{\infty} f_n(x)$

As $\sum \|f_n\| = \sum \frac{n!}{2^{n^2}} < \infty$, so $\sum f_n$ uniformly converges to f. Note that $f^{(n)}(0) = (n!)^2$

The maclaurin series of f is $M(x) = \sum_{n=0}^{\infty} n!\,x^n$, whose the radius of convergence is 0. It means the series only converges at $x=0$.