POW 2017-20

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By integration by part we have

$$\int_0^{\frac{\pi}{2}} \sin^{k+2} x dx = \frac{k+1}{k+2} \int_0^{\frac{\pi}{2}} \sin^k x dx, \int_0^{\frac{\pi}{2}} \sin^0 x dx = \frac{\pi}{2} \tag{1}$$

Then

$$\frac{1}{2^{2n}} \binom{2n}{n} = \prod_{k=1}^{n} \frac{2k-1}{2k} = \frac{2}{\pi} \int_0^{\frac{\pi}{2}} \sin^{2n} x dx \tag{2}$$

For $x \in \left[0, \frac{\pi}{2}\right]$, $\sin x \geq \frac{2}{\pi} x$ so,

$$\frac{2}{\pi} \int_0^{\frac{\pi}{2}} \sin^{2n} x dx \geq \frac{2}{\pi} \int_0^{\frac{\pi}{2}} \left(\frac{2}{\pi} x\right)^{2n} dx = \frac{1}{2n+1} \tag{3}$$

Since $\sum_{n=1}^{\infty} \frac{1}{2n+1}$ diverge, the sum

$$\sum_{n=0}^{\infty} \frac{1}{2^{2n}} \binom{2n}{n} \tag{4}$$

diverge.