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Let $n = 2k + 1$. Also, let $\mathbf{r}_i$ and $\mathbf{c}_i$ denote the ith row vector and the ith column vector, respectively. Since the ith row vector of $A + A^T$ is $\mathbf{r}_i + \mathbf{c}_i$, we have

$$2k + 1 = \text{rank}(A + A^T) \leq \text{rank}(A) + \text{rank}(A^T) = 2\text{rank}(A).$$

Therefore, $\text{rank}(A) \geq k + 1$, and the same result holds for B. Then $\text{nullity}(A) < k + 1$ and $\text{rank}(B) \geq k + 1$, implying that $AB \neq 0$.