POW 22

2015. 국윤범

Let $L_n := (l_{ij}) = \begin{cases} \binom{2i-1}{i-1+j} & \text{if } i \geq j \\ 0 & \text{o.w.} \end{cases}$, which is a lower triangular matrix.

and $U_n := L_n^T$. Note that $(L_nU_n)_{ij} = \sum_{k=1}^{2i-1} \binom{2i-1}{k}\binom{2j-1}{k+j-i}$ for $i \geq j$.

Observe the following identity $\binom{2(i+j-1)}{i+j-1} = \sum_{k=0}^{2i-1} \binom{2i-1}{k}\binom{2j-1}{k+j-i}$.

As $2(L_nU_n)_{ij} = \sum_{k=0}^{2i-1} \binom{2i-1}{k}\binom{2j-1}{k+j-i}$, so $2(L_nU_n)_{ij} = a_{ij}$, where $M_n = (a_{ij})$

Hence, $M_n = (2L_n) \cdot U_n$, so $\det M_n = \det(2L_n) \det U_n = 2^n \cdot \det L_n \det U_n = 2^n$.