

POW 2016-18

2014**** Lee, Jongwon

After adding a big constant to all numbers, we can assume that all numbers are nonnegative. Let the numbers be a_1, \dots, a_m , where $m = 2n + 1$. We shall induct on $S = a_1 + \dots + a_m$.

If S is zero, then all numbers are necessarily zero, so we are done.

Now suppose that the statement holds for all cases with smaller sum than S . By the assumption, $S - a_i$ is the sum of two identical numbers for each i . In particular, $S - a_i$ is even for all i , so that all numbers a_i have the same parity.

If all numbers are even, we can divide everything by 2 and apply the induction hypothesis. If all numbers are odd, then all numbers are at least 1, so we can subtract 1 from all numbers and again apply the induction hypothesis.