

pow 16

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수업

Let $x \in \ker A$, then $x^T(A^T A + A A^T)x = x^T(A + A^T)x$

$$\Rightarrow x^T A A^T x = 0 \Rightarrow \|A^T x\|^2 = 0 \Rightarrow A^T x = 0 \Rightarrow x \in \ker A^T \Rightarrow \ker A \subseteq \ker A^T$$

Similarly, we can show $\ker A^T \subseteq \ker A$. Hence, $\ker A^T = \ker A$

$$\Rightarrow (\ker A^T)^\perp = (\ker A)^\perp \Rightarrow \text{col } A = \text{col } A^T \quad \left(\begin{array}{l} \text{ker } A = (\text{col } A^T)^\perp \\ \text{ker } A^T = (\text{col } A)^\perp \end{array} \right)$$