2016-8 Limit

20160021 Hanpil Kang

$$\lim_{n\to\infty}\cos^{2016}(\pi\sqrt{n^2+4n+9}) \overset{(1)}{=} \lim_{n\to\infty}\cos^{2016}(\pi(\sqrt{n^2+4n+9}-(n+2))$$
$$=\lim_{n\to\infty}\cos^{2016}(\pi(\sqrt{n^2+4n+9}-\sqrt{n^2+4n+4}))$$
$$\overset{(2)}{=} cos^{2016}(\lim_{n\to\infty}\pi(\sqrt{n^2+4n+9}-\sqrt{n^2+4n+4})) \overset{(3)}{=} cos^{2016}0=1$$

(1) $\cos^{2016}$ is periodic function with period π

(2) $\cos^{2016}$ is continuous function. Therefore, order of limit can be changed.

(3) $\lim_{n\to\infty}\sqrt{n^2+4n+9}-\sqrt{n^2+4n+4}=\lim_{n\to\infty}\frac{5}{\sqrt{n^2+4n+9}+\sqrt{n^2+4n+4}}=0$