POW 2016-10 Factorization

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Suppose that A is an $n\times n$ matrix with integer entries and $\det A = p_1^{e_1}p_2^{e_2}\cdots p_k^{e_k}$ for primes $p_1, p_2, \cdots p_k$ and positive integers $e_1, e_2, \cdots e_k$. Prove that there exist $n\times n$ matrices $B_1, B_2, \cdots B_k$ with integer entries such that $A = B_1 B_2 \cdots B_k$ and $\det B_i = p_i^{e_i}$ $(i = 1, 2, \cdots k)$

Sol)

It is enough to prove this lemma.

Lemma: If A is $n\times n$ matrix with integer entries and $\det A = xy$, then there exist two matrices X, Y with integer entries such that $A = XY$, $\det X = x$ and $\det Y = y$.

pf) Let $A = UDV$ be the Smith Normal Form of A, where D is diagonal, U and V are both unimodular. All matrices have integer entries. Since $\det D = xy$ there exists diagonal integer matrices D_1, D_2 $s.t.$ $D = D_1 D_2$ and $\det D_1 = x$, $\det D_2 = y$ So let $X = UD_1$ and $Y = D_2 V$ then $A = UDV = XY$.