Claim : $G = \left\{ \begin{bmatrix} 2^m & 2^n l \\ 0 & 1 \end{bmatrix} : m, n, l \in \mathbb{Z} \right\}$.

proof) Any element of G is a form of $x_1 \cdots x_i$

where $i \geq 0$, $x_1, \cdots, x_i \in \left\{ \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \right\}$.

let $\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} = u$, $\begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix} = u^{-1}$, $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = v$, $\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = v^{-1}$

Since for $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$uA = \begin{bmatrix} 2a & 2b \\ c & d \end{bmatrix}$ $u^{-1}A = \begin{bmatrix} \frac{1}{2}a & \frac{1}{2}b \\ c & d \end{bmatrix}$ $vA = \begin{bmatrix} a+c & b+d \\ c & d \end{bmatrix}$

$v^{-1}A = \begin{bmatrix} a-c & b-d \\ c & d \end{bmatrix}$, one can inductively show that

$G \subseteq \left\{ \begin{bmatrix} 2^m & 2^n l \\ 0 & 1 \end{bmatrix} : m, n, l \in \mathbb{Z} \right\}$. Conversely,

$\begin{bmatrix} 2^m & 2^n l \\ 0 & 1 \end{bmatrix} = u^n v^l u^{m-n} \in G$. ⌟

By Claim, $H = \left\{ \begin{bmatrix} 1 & 2^n l \\ 0 & 1 \end{bmatrix} : n, l \in \mathbb{Z} \right\} \leq G$.

For $A \in H$, let $|A| = \begin{cases} \infty & \text{if } A = I \\ 2^n & \text{if } A \neq I \ \& \ A = \begin{bmatrix} 1 & 2^n l \\ 0 & 1 \end{bmatrix}, 2 \nmid l \end{cases}$

Then $|AB| \geq \min\{|A|, |B|\}$, $|A^{-1}| = |A|$ for $A, B \in H$

But then for any choice $A_1, \cdots, A_k \in H$,

$$\inf\{|A| : A \in H\} = 0 < \underbrace{\inf\{|A| : A \in \langle A_1, \cdots, A_k \rangle\}}_{\begin{array}{c} \text{\rotatebox{90}{$\leq$}} \\ \min\{|A_1|, \cdots, |A_k|\} \end{array}}$$

$\therefore$ H is not f.g.