POW 2015-23

2014**** Lee, Jongwon

Given $x \in [0, 1)$, let $0.a_1a_2\ldots$ be an infinite binary expansion of x, possibly adding zeroes if a binary expansion is finite. We shall say that this expansion if 'nice' if it contains infinitely many zeroes.

Firstly, there is a bijection between all nice binary expansions and $[0, 1)$, since for any binary expansion, if we see $\ldots 011111\ldots$, we can change it to $\ldots 100000\ldots$.

Also, if $x = 0.a_1a_2\ldots$ is a nice binary expansion, $0 \leq 2x < 1$ iff $a_1 = 0$. This is because since x cannot equal $0.111\ldots$, we have $x < 0.11\ldots = 1/2$.

Therefore, it is clear to see that $f(x) = 0.a_2a_3\ldots$, so $f^7(x) = x$ is equivalent to the sequence $\{a_i\}$ having a period 7. This is equivalent to x in the form

$$n\left(2^{-7} + 2^{-14} + \ldots\right) = \frac{n}{127}$$

where $0 \leq n \leq 126$.