POW 2015-21 : Differentiable function

KAIST 수리과학과 14학번 장기정

November 14, 2015

Let us introduce following lemma:

Lemma 1. Any bounded monotone functions in [0,1] is differentiable at almost everywhere.

Proof. more strong form is possible : any real function which is monotone is differentiable almost everywhere. (Proof is available in Theorem 14, in this webpage) □

Let us define $F, G : [0,1] \to \mathbf{R}$ by $F(x) = \sum_{x_r < x} y_r$ and $G(x) = \sum_{x_r \leq x} y_r$. (Since the series $\sum_{n=1}^{\infty} y_n$ (absolutely) converges, we can define F, G as well.) Then we immediately have F, G are bounded monotone functions, hence differentiable at almost everywhere, hence so $f = G - F$ is.