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Sunghyuk Park, 2014[redacted]

(In the following proof, for any $N \times N$ matrix M, $M_\sharp$ denotes the submatrix of M obtained by deleting the N-th row and column.)

Proof. Let $H = LL^*$ be the Cholesky decomposition, and $A = L^{-1}$. $H' = L_\sharp L_\sharp^*$ because L is lower triangular. Also, $G = A^*A$ and $G' = A_\sharp^* A_\sharp$. In this time the upper triangular matrix(A^*) is multiplied in front of the lower triangular matrix(A), so $G_\sharp \neq G'$. Their difference is $G_{ij} - G'_{ij} = A^*_{iN} A_{Nj}$. Since $G_{iN} = A^*_{iN} A_{NN}$, $G_{Nj} = A^*_{NN} A_{Nj}$ and $G_{NN} = A^*_{NN} A_{NN}$, we have

$$G_{ij} - G'_{ij} = \frac{G_{iN} G_{Nj}}{G_{NN}}$$

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