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Let $g(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$ be the normal distribution with mean 0 and standard deviation 1. Also, for an arbitrary nonnegative number $t$, consider $h(y) = t \log y - y$. Since $h'(y) = \frac{t}{y} - 1$, it is easy to see that $h(y) \leq h(t) = t \log t - t$. Then, for all $x \in \mathbb{R}$, we have

$$f(x) \log g(x) - g(x) \leq f(x) \log f(x) - f(x).$$

Integrating both sides, since $\int f(x)dx = \int g(x)dx = 1$, we have

$$\begin{aligned} \int f(x) \log f(x) dx &\geq \int f(x) \log g(x) dx \\ &= \int f(x) \left( -\frac{1}{2} \log 2\pi - \frac{x^2}{2} \right) \\ &= -\frac{1}{2} \log 2\pi - \frac{1}{2}. \end{aligned}$$

The equality holds obviously iff $f = g$.