POW 2015-10

2012[redacted] 이영민

$$\sum_{i=1}^{n} w_i = 1, \quad x_1, x_2, ...x_n \in [0, \pi], \text{ then } \sin(\prod_{i=1}^{n} x_i^{w_i}) \geq \prod_{i=1}^{n} (\sin x_i)^{wi}$$

If $\exists i$, $x_i = 0$ or $x_i = \pi$, then above inequality is trivial.
So we consider $\forall i$, $x_i \in (0, \pi)$.
Let $f(x) = \ln(\sin(e^x))$, $x \in (-\infty, \ln \pi)$. (Well-defined) Then,

$$f''(x) = \frac{e^x(\cos(e^x)\sin(e^x) - e^x)}{(\sin(e^x))^2} \leq 0$$

(substitution $e^x = t, \quad t \in (0, \pi)$, $\cos t \sin t - t = \frac{\sin 2t}{2} - t \leq 0$)
$\forall i, x_i \in (0, \pi)$ so, $\ln x_i \in (-\infty, \ln \pi)$
By Jensen's inequality,

$$\ln(\sin(\prod_{i=1}^{n} x_i^{w_i})) = \ln(\sin(e^{\sum_{i=1}^{n} w_i \ln x_i})) \geq \sum_{i=1}^{n} w_i \ln(\sin(e^{\ln x_i})) = \ln(\prod_{i=1}^{n} (\sin(x_i))^{w_i})$$

Therefore,

$$\sin(\prod_{i=1}^{n} x_i^{w_i}) \geq \prod_{i=1}^{n} (\sin(x_i))^{w_i}$$