POW 2015-8 201[redacted] 이수철

existence of $A \subset \mathbb{R}^2$ s.t. $|A \cap L| = 2$ for every straight line L

Pf let $\{L_x : x < c\}$ be enumeration of every straight line in $\mathbb{R}^2$

use transfinite induction

suppose for x, $x < c$, there is $\{A_y : y < x\}$

s.t. ① $|A_y| \le 2$

② for all line L in $\mathbb{R}^2$, $\left|\left(\bigcup_{z \le y} A_z\right) \cap L\right| \le 2$

③ $\left|\left(\bigcup_{z \le y} A_z\right) \cap L_y\right| = 2$

let $A^\circ_x = \bigcup_{y<x} A_y$, $\mathcal{L}^\circ_x$ union of lines generated by every pair of points of A°_x

notice that $|A^\circ_x|^2 < c$, ($\because |A^\circ_x| < c$ by ①)

$|A^\circ_x \cap L_x| \le 2$

If $|A^\circ_x \cap L_x| = 2$, let $A_x = \emptyset$.

If $|A^\circ_x \cap L_x| = 1$, $\exists a \in L_x \setminus \mathcal{L}^\circ_x$ let $A_x = \{a\}$ (If $|A^\circ_x \cap L_x| \le 1$, $|\mathcal{L}^\circ_x \cap L_x| \le |A^\circ_x|^2 < c$)

If $|A^\circ_x \cap L_x| = 0$, $\exists a, b \in L_x \setminus \mathcal{L}^\circ_x$, $a \ne b$ let $A_x = \{a, b\}$

then A_x satisfies ①, ②, ③

$\therefore$ by transfinite induction, we can take $A = \bigcup_{x<c} A_x$

and this is the set, $|A \cap L| = 2$ for every straight line L