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2014**** Lee, Jongwon

There always exists an integer $n_k \in [2k\pi + \frac{\pi}{6}, 2k\pi + \frac{5\pi}{6}]$ for all $k \in \mathbb{N}$, so that $\frac{1}{|n_k \sin n_k|} \leq \frac{2}{|n_k|} \to 0$.

However, using continued fraction, or by referring to Hurwitz's theorem, there exists a sequence of reduced fractions $\{\frac{m_k}{l_k}\}$ such that $|\frac{m_k}{l_k} - 2\pi| < \frac{1}{l_k^2}$. But then, we have

$$\frac{1}{|m_k \sin m_k|} = \frac{1}{|m_k \sin(m_k - 2l_k\pi)|} > \frac{1}{|m_k(m_k - 2l_k\pi)|} > \frac{l_k}{m_k} \to \frac{1}{2\pi}$$

proving that $\frac{1}{n \sin n}$ does not converge.