POW2015-5 Solution

KAIST 11학번 수리과학과 최두성

Definition $M = ({}^i1^j)$ be a matrix in $F^{n\times n}$ whose entries are all zero except $(M)_{ij} = 1$.

proof

For $i \neq k$, $tr[A({}^i1^j)({}^j1^k)] = tr[A({}^i1^k)] = A_{ki}$, but $tr[A({}^j1^k)({}^i1^j)] = 0$ $(\because ({}^j1^k)({}^i1^j) = O)$

$\therefore A_{ki} = 0$ for all $i \neq k$

Then for $i \neq j$, $tr[A({}^i1^j)({}^j1^i)] = tr[A({}^i1^i)] = A_{ii}$ and $tr[A({}^j1^i)({}^i1^j)] = tr[A({}^j1^j)] = A_{jj}$

$\therefore A_{ii} = A_{jj}$ for all $i \neq j$

We can check that $A = cI$ satisfies $tr(AXY) = tr(AYX)$ for all X, Y

since the trace function is commutative.

Therefore, $A = cI$ for any c in F.