# POW 2015-1

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Let $\omega = e^{2\pi i/2^n}$ be the primitive $2^n$-th root of unitiy. Also, for a subset $X$ of $A$, define $S_X$ as $\sum_{x\in X} x$. If $S_X$ are different modulo $2^n$ for all subsets $X$, since there are $2^n$ subsets in total, we have

$$\prod_{a\in A}(1+\omega^a) = \sum_{X\subseteq A} \omega^{S_X} = \sum_{i=0}^{2^n-1} \omega^i = 0$$

so that $1+\omega^a = 0$ for some $a \in A$. But this implies that $a \equiv 2^{n-1} \pmod{2^n}$, contradicting that $a$ is odd.