$$f_n(x) = \prod_{k=2}^{n} \frac{(k+x-1)(k+x)}{k^2}$$

$$= \prod_{k=2}^{n} \frac{(k+x-\frac{1}{2})^2 - \frac{1}{4}}{k^2}$$

(a) If $x < \frac{1}{2}$, $f_n(x) \leq \prod_{k=2}^{n} \left((k-\varepsilon)/k\right)^2$.

Since $\ln(1-\varepsilon/k) = -\varepsilon/k + o(1/k)$, $f_n(x) \to 0$.

(b) If $x > \frac{1}{2}$, $f_n(x) \geq \frac{1}{2} \prod_{k=2}^{n} \left((k+\varepsilon)/k\right)^2$.

Similarly, $\ln(1+\varepsilon/k) = \varepsilon/k + o(1/k)$, $f_n(x) \to \infty$.

(c) If $x = \frac{1}{2}$, from $\sin(\pi x) = \pi x \prod_{k=1}^{\infty} (k^2 - x^2)/k^2$

we have $f_n(x) \to \frac{2}{\pi} \cdot \sin(\frac{\pi}{2}) \cdot \frac{4}{3} = 8/3\pi$.