

for $a, b \in G$

$$a^2 = (abb^{-1})^2 = (cab)b^{-1})^2 \\ = (b^{-1}ab)^2 = b^{-1}a^2b$$

i.e. $a^2b = ba^2$ for any $a, b \in G$ (*)

Let $c = aba^{-1}b^{-1}$

similar to above

$$c^2 = abc^2(ab)^{-1}$$

$$= ab(aba^{-1}b^{-1}aba^{-1}b^{-1})(ab)^{-1}$$

$$= (ab)^2(a^{-1}b^{-1}ab a^{-1}b^{-1})(ab)^{-1}$$

$$= a^{-1}ab^2b^{-1}ab a^{-1}b^{-1}(ab)^{-1} \quad (\text{by } (*))$$

$$= ba^2ba^{-1}b^{-1}(ab)^{-1}$$

$$= b^2ab^{-1}(ab)^{-1} \quad (\text{by } (*))$$

$$= ab^2b^{-1}(ab)^{-1} = (ab)(ab)^{-1} = e$$

Since G has no element of order 2,

we can deduce $c = e$.

$$\text{Thus, } ab(a^{-1}b^{-1}) = e$$

$$\text{i.e. } ab = ba \text{ for all } a, b \in G$$

G is abelian.