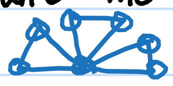
NOTE We need to count the zero set.
Otherwise,  can be a counterexample.

SOLUTION

Let S_i be the number of independence set of size i.
Define $I(G;x) \equiv \sum_{i=0}^{\alpha(G)} S_i x^i$. $(I(\phi;x)=1)$

Prop 1 For $u,v \in V(G)$, $uv \in E(G)$,

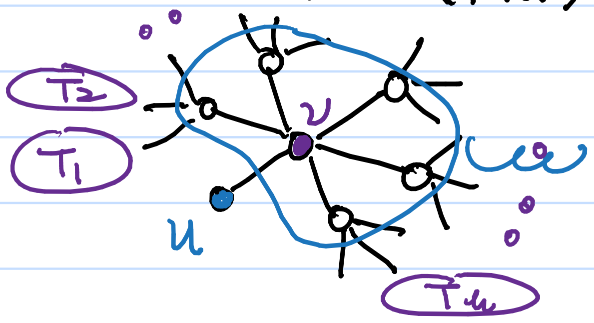
1) $I(G;x) = I(G\setminus v;x) + x\,I(G\setminus v\setminus N(v);x)$
2) $I(G;x) = I(G\setminus uv;x) - x^2 I(G\setminus N(u)\setminus N(v);x)$

$\because$ 1) Consider independent sets with/without v.
2) Consider independent sets containing both u,v in $G\setminus uv$. I will ommit the details.

Prop 2 For a tree T, $I(T;-1) \in \{-1,0,1\}$.

$\because$ Induction on $n=|V(T)|$. For $n=1$, $I(T;-1)=0$.
Suppose that Prop 2 holds for any $k<n$ with some $n \geq 2$. Consider any tree with n vertices. Choose $u \in V(T)$ with $\deg(u)=1$, and let $N(u)=\{v\}$.
By Prop 1,

$$I(T;x) = I(T\setminus uv;x) - x^2 I(T\setminus N(u)\setminus N(v);x)$$
$$= (1+x)\,I(T\setminus u;x) - x^2 I(T\setminus N(v)\setminus v;x)$$



Note that $T\setminus N(v)\setminus v = T_1 \cup \cdots \cup T_k$ for some smaller trees (possibly an empty graph). Hence,

$$I(T\setminus N(v)\setminus v;x) = \prod_{i=1}^{k} I(T_i;x).$$

It follows that, from the induction hypothesis,

$$I(T;-1) = -\prod_{i=1}^{k} I(T_i;-1) \in \{-1,0,1\}.$$

CLAIM Let G be a simple graph, and k_G be the smallest number of vertices required to delete every cycle. Then, $|I(G;-1)| \leq 2^{k_G}$.

$\because$ Induction on k_G. If $k_G=0$, then G is a forest; thus, $I(G;-1) \in \{-1,0,1\}$ by Prop 2. Suppose that $|I(G;-1)| \leq 2^{k_G}$ for any G with $k_G < k$ for some $k>0$.

Assume that G is given and $k_G = k$. Let $\{v_1,\cdots,v_k\} \subseteq V(G)$ be the k vertices deleting all cycles. By the minimality, all v_i are contained in a cycle. Hence,

$$I(G;x) = I(G\setminus v_1;x) + x\,I(G\setminus v_1\setminus N(v_1);x)$$

and $k_{G\setminus v_1} = k-1$, $k_{G\setminus v_1\setminus N(v_1)} \leq k-1$. Therefore, by the induction hypothesis,

$$|I(G;-1)| = |I(G\setminus v_1;-1) - I(G\setminus v_1\setminus N(v_1);-1)|$$
$$\leq |I(G\setminus v_1;-1)| + |I(G\setminus v_1\setminus N(v_1)|$$
$$\leq 2^{k-1} + 2^{k-1} = 2^k,$$

and the claim follows. ∎