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The following is a theorem by Sylvester and Schur.

Theorem ([1]) If n, k are positive integers satisfying $n > k$, then, in the set of integers $n, n+1, n+2, \ldots, n+k-1$, there is a number containing a prime divisor greater than k.

Back to the main problem,

Solution Let $x\%y$ denote the remainder after division of x by y. First, if $a < b$, we can take a prime p greater than b, so that $a\%p = a < b = b\%p$.

Secondly, if $a \geq 2b$, according to the theorem of Sylvester and Schur, among $a-b+1, a-b+2, \ldots, a$, there is a number $a-t$ containing a prime divisor p such that $0 \leq t < b < p$. But then, $a\%p = t < b = b\%p$.

Lastly, if $2b > a > b$, according to the theorem of Sylvester and Schur, among $b+1, b+2, \ldots, a$ there is a number $b+t$ containing a prime divisor p such that $1 \leq t \leq a-b < p$. Again, since $a \equiv a-b+b \equiv a-b-t \pmod{p}$, we have $a\%p = a-b-t < p-t = b\%p$.

Reference

[1] P. Erdős, "A Theorem of Sylvester and Schur". *Journal of the London Mathematical Society*, **9** (1934), pp. 282-288.