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Solution. We show that $(m, n) = (t^2 + t, (t^2 + t + 1)^3)$ satisfies the problem where t is any positive integer greater than 1. Clearly, there are infinitely many of them, m, n are positive, and $\gcd(m, n) = 1$ because $\gcd(t^2 + t, t^2 + t + 1) = 1$. Also,

$$(x + m)^3 - nx = (x - 1)(x - t^3)(x + (t + 1)^3)$$

and since $1, t^3, -(t + 1)^3$ are distinct integers, this completes the proof.