Proof. For each $x \in A = \{(a_1, \cdots, a_n) : a_i = \pm 1 (i = 1, \cdots, n)\}$, let N_x be the set of all points in A that differ from x in exactly one coordinate. It is obvious that $|N_x| = n$ for any $x \in A$.

Now define $B := \{(x, y) : x \in X, y \in N_x \subset A\}$. Then $|B| = \sum_{x \in X} |N_x| = n|X| > 2^{n+1}$. Since $|A| = 2^n$, second coordinate of an element in B must be one of 2^n points in A. Then by pigeonhole principle, there is a point $y \in A$ such that there exist x_1, x_2, x_3 satisfying $y \in N_{x_i}$ for $i = 1, 2, 3$. These are three points in X that differ from y in exactly one coordinate. Thus these points differ from each other in exactly two coordinate, which means that these points form an equilateral triangle.

$\square$