POW 2012-18

2010
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October 8, 2012

Ler $S = \{r_n | n \in N\}$. Let's assume that $x = 0.a_{1,1}a_{2,2}\cdots$ is a rational number. Then, $\exists$ a minimum $N \in \mathbb{N}$ and $\exists T \in \mathbb{N}$ s.t $a_{i,i} = a_{i+T,i+T}$ $\forall i > N$.
Now, define a sequence $\{x_n\}$ s.t $x_n = 0.b_{1,1}b_{2,2}\cdots$
where

$$b_{i,i} = \begin{cases} 0 & a_{i,i} \neq 0, i \leq N+n \\ 1 & a_{i,i} = 0, i \leq N+n \\ 2 & a_{i,i} \neq 2, i > N+n \\ 3 & a_{i,i} = 2, i > N+n \end{cases}$$

It is easy to see that x_n's are all rational numbers.
Morevoer, $x_n \notin S$ for all $n \in \mathbb{N}$. But, It is contradiction. Since, S contains all rational numbers except finitely many raional numbers.