

KAIST POW 2012-16

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(A finite ring) Prove that if a finite ring has two elements x and y such that $xyy = y$, then $yxxy = y$.

Proof. Consider a subset $\{y^k | k \in \mathbb{N}\}$. Since the ring is finite, the subset is also finite, so there must exist $n, m \in \mathbb{N}$ such that $y^n = y^{n+m}$. If $n > 1$,

$$y^{n-1} = xy^n = xy^{n+m} = y^{n-1+m}$$

By repeating this, we get $y = y^{m+1}$. Therefore,

$$yxxy = yxy^{m+1} = yy^m = y^{m+1} = y$$