# POW2011-24 - $(n-k)$ choose $k$

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Let $g(x)$ be a generating function of the given sum. then

$$g(x) = \sum_{n \geq 0} \left( \sum_{k \geq 0} \binom{n-k}{k} (-4)^{n-k} \right) x^n$$

$$= \sum_{n \geq 0} \left( \sum_{k \geq 0} \binom{k}{n-k} (-4)^k \right) x^n$$

$$= \sum_{k \geq 0} (-4)^k x^k \left( \sum_{n \geq 0} \binom{k}{n-k} x^{n-k} \right)$$

$$= \sum_{k \geq 0} (-4)^k x^k (1+x)^k$$

$$= \frac{1}{1 + 4x + 4x^2} = \frac{1}{(1+2x)^2} = \frac{-1}{2} \frac{d}{dx} \left( \frac{1}{1+2x} \right)$$

$$= \sum_{n \geq 0} (n+1)(-2x)^n$$

Therefore, the given sum is exactly $(n+1)(-2)^n$.