SOLUTION FOR POW 2011-16

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If $|N| \leq \left|\bigcup_{i\in N} A_i\right|$ holds for every $N \subseteq \{1, 2, \ldots, n\}$, then we can find distinct $a_1, a_2, \ldots, a_n$ such that $a_i \in A_i$ by Hall's theorem. Without loss of generality, we may assume $N = \{1, 2, \ldots, k\}$.

Let $\bigcup_{i\in N} A_i = \{x_1, x_2, \ldots, x_t\}$ and define a $k \times t$ matrix M over $\mathbb{F}_2$ such that $M_{i,j} = 1$ if and only if A_i contains x_j. Then it is easy to see that $X = MM^T$ is the $k \times k$ identity matrix. Therefore the rank of M is k, which implies $k \leq t$ as desired.