2011-5 Linear function on matrices

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Let A_{ij} be a matrix such that $A_{ij} = (a_{kl}),\ a_{kl} = 0\ for\ (k,l) \neq (i,j)\ , a_{ij} = 1.$

$A_{ij} \times A_{ii} = A_{ij}$ and $A_{ii} \times A_{ij} = 0$(zero matrix).

$A_{ij} \times A_{ji} = A_{jj}$ and $A_{ji} \times A_{ij} = A_{ii}$.

So, $f(A_{ij}) = f(0) = 0$. $f(A_{ii}) = f(A_{jj})\ for\ all\ (i,j)\ and\ i \neq j$. By property of linear function.

Let $f(A_{11}) = c$. C is a scalar. Then,

$f(A) = \sum f(a_{ij}A_{ij})$ (a_{ij} is an entry of A).

$= \sum f(a_{ii}A_{ii}) = \sum a_{ii} f(A_{ii}) = c \sum a_{ii} = c\ tr(A)$. (tr(A) is a trace of A).

We know that tr(AB) = tr(BA).

Therefore, Solution : $f(A) = c\ tr(A)$ (c is scalar).