POW 2011-4

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Without loss of generality, we assume that the coefficient of x^n is positive.
(proof is similar when the coefficient is negative.)
Then $f(x) = a(x-r_1)(x-r_2)\cdots(x-r_n)$ with $a>0$.
By computing, $f'(x) = (x-r_1)(x-r_2)\cdots(x-r_{n-2})\{(x-r_{n-1})+(x-r_n)\}$
$+\sum_{i=1}^{n-2}\frac{(x-r_1)(x-r_2)\cdots(x-r_{n-2})}{x-r_i}(x-r_{n-1})(x-r_n)$

It is obvious that $\forall i=1,\cdots,n-2,\ \frac{r_n+r_{n-1}}{2}-r_i>0$.

Thus, $\frac{(x-r_1)(x-r_2)\cdots(x-r_n)}{x-r_i}>0$ when $x=\frac{r_n+r_{n-1}}{2}$ (Let $r=\frac{r_n+r_{n-1}}{2}$)

$\therefore\ \sum_{i=1}^{n-2}\frac{(r-r_1)(r-r_2)\cdots(r-r_{n-2})}{r-r_i}(r-r_{n-1})(r-r_n)<0\quad(\because r_{n-1}<r<r_n)\quad\cdots①$

Also, $(r-r_1)(r-r_2)\cdots(r-r_{n-2})\{(r-r_{n-1})+(r-r_n)\}$
$=(r-r_1)(r-r_2)\cdots(r-r_{n-2})(2r-r_{n-1}-r_n)=0\quad\cdots②$

Therefore, $f'(r)=(r-r_1)(r-r_2)\cdots(r-r_{n-2})(2r-r_n-r_{n-1})$
$+\sum_{i=1}^{n-2}\frac{(r-r_1)(r-r_2)\cdots(r-r_{n-2})}{r-r_i}(r-r_{n-1})(r-r_n)$ (by ②)
$=\sum_{i=1}^{n-2}\frac{(r-r_1)(r-r_2)\cdots(r-r_{n-2})}{r-r_i}(r-r_{n-1})(r-r_n)<0$ (by ①)

Since r_n is a zero which has maximum value, $f'(r_n)>0$.
(It is easily observed by a graph of f)
Thus there exist q' such that $r_{n-1}<r<q'<r_n$ and $f'(q')=0$.
Because there are exact one zero of f' between r_{n-1} and r_n, $q=q'$

Therefore, $q=q'>r=\frac{r_1+r_2}{2}$.

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