POW 2010-16

[redacted]

정성구

Let $n=p^e$ for some prime p, $e\in\mathbb{N}$

then divisors of n are $1, p, p^2, \cdots, p^e$

If $p=3$, $D(n,1)=1$, $D(n,2)=0$

If $p\equiv 1 \pmod 3$, $p^i\equiv 1 \pmod 3$ for $0\le i\le e$ $\Rightarrow$ $D(n,1)=e+1$, $D(n,2)=0$

If $p\equiv -1 \pmod 3$, $p^{2i}\equiv 1 \pmod 3$ for $0\le i\le [\frac{e}{2}]$, $p^{2i+1}\equiv -1 \pmod 3$ for $0\le i\le [\frac{e-1}{2}]$

$\Rightarrow D(n,1)=[\frac{e+2}{2}]$, $D(n,2)=[\frac{e+1}{2}]$

So, $D(n,1)\ge D(n,2)$

Let $n=ab$, $(a,b)=1$ and $a_1,\cdots,a_s$ be all divisors of a, $b_1,\cdots,b_t$ be all divisors of b. Then all divisors of ab are a_ib_j $(1\le i\le s,\ 1\le j\le t)$

It is easy to see that

$D(n,1)=D(a,1)D(b,1)+D(a,2)D(b,2)$

$D(n,2)=D(a,1)D(b,2)+D(a,2)D(b,1)$

$\Rightarrow D(n,1)-D(n,2)=(D(a,1)-D(a,2))(D(b,1)-D(b,2))$

Therefore, $D(n,1)\ge D(n,2)$, provided that $D(a,1)\ge D(a,2)$, $D(b,1)\ge D(b,2)$

In general, $n=p_1^{e_1}\cdots p_r^{e_r}$ (prime factorization)

$D(p_i^{e_i},1)\ge D(p_i^{e_i},2)$ for each i and $(p_i^{e_i}, p_j^{e_j})=1$ if $i\ne j$ so by induction, $D(n,1)\ge D(n,2)$