POW2010-2 Nonsingular Matrix

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Let $A=(a_{ij})$ be an n×n matrix of complex numbers such that $\sum_{j=1}^{n} |a_{ij}| < 1$ for each i. Prove that I-A is nonsingular.

Proof)

Suppose that I-A is singular.
Then det(I-A)=0, and $\lambda=1$ is an eigenvalue of A.
Let **x** be a nonzero vector in C^n which is an eigenvector of A corresponding to $\lambda=1$.
Then **x** satisfies that A**x**=**x**.
Then,

$$\sum_{j} a_j\, x_j = x_i \qquad (*)$$

for each i.
Let x_k be the k^{th} component of **x** such that $|x_k| \geq |x_i|$ for $i \in \{1,2,\cdots,n\}$.($k \in \{1,2,\cdots,n\}$)
Since

$$\left|\sum_{j} a_{kj} x_j\right| \leq \sum_{j} |a_{kj} x_j| = \sum_{j} |a_{kj}||x_j| \leq |x_k| \sum_{j} |a_{kj}| < |x_k| \quad \text{(by (1) and triangular inequality)}$$

and by the equality (*),

$$\left|\sum_{j} a_{kj} x_j\right| = |x_k|$$

there is a contradiction.

Therefore, I-A is nonsingular.

QED

(1) Let e=a+bi, f=c+di be complex numbers.(a,b,c,d are real numbers)
$|ef|=|(ac-bd)+(ad+bc)i|=((ac-bd)^2+(ad+bc)^2)^{1/2}=((a^2+b^2)(c^2+d^2))^{1/2}=|e||f|$